DISTRIBUTION NORMAL CONTACT STRESSES IN THE ROLL GAP AT A CONSTANT SHEAR STRESS

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Received: 10.03.2015
Accepted: 21.03.2015

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Abstract
Differential equation describing stress state in the rolling gap was derived first time by Karman. Since the solution of the differential equation is not easy many authors try to simplify of entry conditions. Some authors have replaced the circular arch of the contact zone of rolls by straight line, polygonal curve or parabola for simplifications of solution. These simplifications allow to obtain analytical solution differential equation but with acceptation some inaccuracy of the final results. Another solution of the differential equation was focused on the substitution of the analytical solution by the numerical solution, but should also expect some uncertainty of the final results. A more sophisticated solution was given by Gubkin is based on defining a constant shear stress and the approximation of the circular arch through the straight line. Gubkin for analytic solution of differential equation used one constant that includes the friction coefficient and second constant which is including the geometry of the rolling gap. The contribution of this paper is an original analytical solution of the differential equation based on the description of the contact arc by the equation of a circle. The proposed solution for the calculation of normal stress distribution is described by two constants. The first constant is describing the geometry of the rolling gap and the second describes friction coefficient. The final solution of differential equation is sum of two independent functions involving the shear stress as a variable value. The proposed solution does not consider with material work hardening during processing.

Keywords: theory of rolling, constant shear stress, differential equations, normal stress, relative stress

1 Introduction

The process of lengthwise rolling can be described as the action of active forces onto rolling direction with the consideration of equilibrium conditions for the element. Theory of lengthwise rolling process was presented for the first time by von Karman [1] in 1925. This theory described the equilibrium conditions of the element in the rolling gap by two-dimensional differential equation. The geometrical characterization of the differential element is represented in **Fig. 1**.

The horizontal projection of all the forces acting on the element must be in equilibrium state. On the base of the sum of the horizontal active forces was derived differential equation of contact stresses for two – dimensional deformation in the consideration of the forward and the backward slip zones. The procedure for derivation of the two-dimensional differential equation can be found in the classical literature of rolling Počta [2], Avitzur [3], Hensel and Spittel [4] and

DOI 10.12776/ams.v21i1.549 p-ISSN 1335-1532 e-ISSN 1338-1156

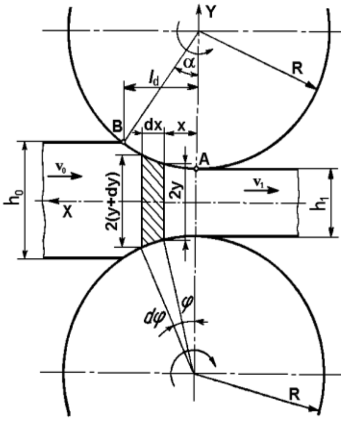

Fig. 1 Determination of geometric relationships

Mielnik [5]. More recent literature are the publications Hajduk and Konvičný [6], Kollerová et al [7] and Pernis [8]. The stress state in rolling gap describes differential equation:

$$\frac{d\sigma_x}{dx} - \frac{\sigma_n - \sigma_x}{y} \cdot \frac{dy}{dx} \pm \frac{\tau}{y} = 0 \tag{1.}$$

where:
- plus sign (+) is for backward slip zone
- minus sign (–) is for forward slip zone
- σ_n - normal contact stress on rolls
- τ - shear stress between rolls and rolling material
- x, y - coordinates of the cylinder touching the rolled material

The following Tresca condition of plasticity is used in eq.(1):

$$\sigma_n - \sigma_x = \sigma_a \tag{2.}$$

where:
- $\sigma_y \doteq \sigma_n$ – maximal principal stress (vertical direction)
- σ_x – minimal principal stress (horizontal direction)
- σ_a - stress which represents real deformation resistance [9–12]

The shear stress can be determined from on the base two following assumptions:
- the shear stress is varied and also is proportional to the normal contact stress on rolls:

$$\tau = f \cdot \sigma_n \tag{3.}$$

- the shear stress is constant and also is proportional to the real deformation resistance:

$$\tau = f \cdot \sigma_a \tag{4.}$$

Substituting eq. (2) and eq.(4) to the eq.(1) is obtained the following form of two-dimensional differential equation with a single variable that is stress:

$$\frac{d\sigma_n}{dx} - \frac{\sigma_a}{y} \cdot \frac{dy}{dx} \pm \frac{\sigma_a}{y} \cdot f = 0 \tag{5.}$$

DOI 10.12776/ams.v21i1.549 p-ISSN 1335-1532
e-ISSN 1338-1156

The geometry of the roll which is in contact with the rolling material can be described by the coordinates x and y wherein the variable y is a function of the coordinate x. The eq.(5) is characterized by two constants: actual deformation resistance and friction coefficient. Assuming that the material during the rolling does not exhibit work hardening it can be the differential equation divided by the actual deformation resistance σ_a and used new variable which is relative normal contact stress $\bar{\sigma}_n$:

$$\bar{\sigma}_n = \frac{\sigma_n}{\sigma_a} \tag{6.}$$

where: - $\bar{\sigma}$ – is a sigma function
Substituting eq. (6) to the eq.(5) is obtained formula in which is eliminating the actual deformation resistance σ_a:

$$\frac{d\bar{\sigma}_n}{dx} - \frac{1}{y} \cdot \left(\frac{dy}{dx} \pm f \right) = 0 \tag{7.}$$

Using this equation (eq.(7)) the solution becomes independent of the properties of the rolled material.

2 Solution of differential equation according to Gubkin

The analytical solutions of eq.(1) are based on certain assumptions and mainly simplifications. Gubkin [13] approximated the circular arc with a parabola and also by a straight line as shown in **Fig. 2**.

$$y = \frac{\Delta h}{2l_d} \cdot x + \frac{h_1}{2} \tag{8.}$$

$$\frac{dy}{dx} = \frac{\Delta h}{2l_d} \tag{9.}$$

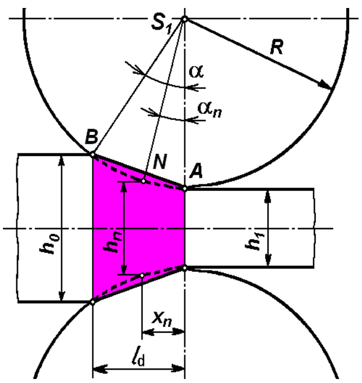

Fig. 2 The Contact Arc according to Gubkin

Substituting eq. (9) to the eq.(7) is obtained following form:

DOI 10.12776/ams.v21i1.549 p-ISSN 1335-1532
e-ISSN 1338-1156

$$\frac{\frac{d\bar{\sigma}_n}{2l_d}}{\Delta h} - \left(\frac{\Delta h}{2l_d} \pm f\right) \cdot \frac{dy}{y} = 0 \quad (10.)$$

and after simplification:

$$d\bar{\sigma}_n = (1 \pm m) \cdot \frac{dy}{y} \quad (11.)$$

where: $\mathrm{m} = 2\mathrm{f} \cdot \frac{\mathrm{l_d}}{\Delta \mathrm{h}}$ (12.)

By integration of the eq.(11) will be obtained the inscription for the backward slip zone:

$$\bar{\sigma}_{nB} = (1 - m) \cdot \ln y + C_B \quad (13.)$$

and for the forward slip zone:

$$\bar{\sigma}_{nF} = (1 + m) \cdot \ln y + C_F \quad (14.)$$

The integration constant C_B for the backward slip zone and C_F for the forward slip zone were specified from the boundary conditions of the material input and exit into resp. from rolling gap. It is assumed that the lengthwise rolling process is realized without forward and backward stretching forces, without material hardening during plastic deformation and without roll flattening. The vertical coordinates of points **A** and **B** according to **Fig. 1** are as follows: point **A**: $y=h_1/2$, point **B**: $y=h_0/2$ and horizontal stress in these points is σ_x=0. To apply $\sigma_n = \sigma_a$ must be valid a condition of plasticity. From eq.(13) is determined integration constant for the backward slip zone where is valid: $\bar{\sigma}_{nB} = 1$ a $y=h_0/2$:

$$C_B = 1 - (1 - m) \cdot \ln \frac{h_0}{2} \quad (15.)$$

Similarly from eq.(14) is determined integration constant for the forward slip zone where is valid: $\bar{\sigma}_{nF} = 1$ a $y=h_1/2$

$$C_F = 1 - (1 + m) \cdot \ln \frac{h_1}{2} \quad (16.)$$

The equation describing of the distribution of the relative normal contact stress for the forward slip zone can be written as follows:

$$\bar{\sigma}_{nB} = 1 + (1 - m) \cdot \ln \frac{2y}{h_0} \quad (17.)$$

and the equation for the calculation of the relative normal contact stress for the backward slip zone can be written as follows:

$$\bar{\sigma}_{nF} = 1 + (1 + m) \cdot \ln \frac{2y}{h_1} \quad (18.)$$

DOI 10.12776/ams.v21i1.549 p-ISSN 1335-1532
e-ISSN 1338-1156

The coordinate y is described by eq.(8). To calculate the relative normal contact stress the coordinate y is replaced by the relative coordinate x/l_d.

$$\frac{2y}{h_0} = 1 - \varepsilon\left(1 - \frac{x}{l_d}\right) \tag{19.}$$

or:

$$\frac{2y}{h_1} = 1 + \frac{\varepsilon}{1-\varepsilon} \cdot \frac{x}{l_d} \tag{20.}$$

Substituting eq. (19) to the eq.(17) is obtained the following form for the relative normal contact stress for the backward slip zone:

$$\bar{\sigma}_{nB} = 1 + (1-m) \cdot \ln\left(1 - \varepsilon\left(1 - \frac{x}{l_d}\right)\right) \tag{21.}$$

Substituting eq. (20) to the eq.(18) is obtained the following form for the relative normal contact stress for the forward slip zone:

$$\bar{\sigma}_{nF} = 1 + (1+m) \cdot \ln\left(1 + \frac{\varepsilon}{1-\varepsilon} \cdot \frac{x}{l_d}\right) \tag{22.}$$

The visualization of the rolling equation eq.(21) and eq.(22) in depend on the relative coordinate x/l_d and constant shear stress is given in **Fig. 3**.

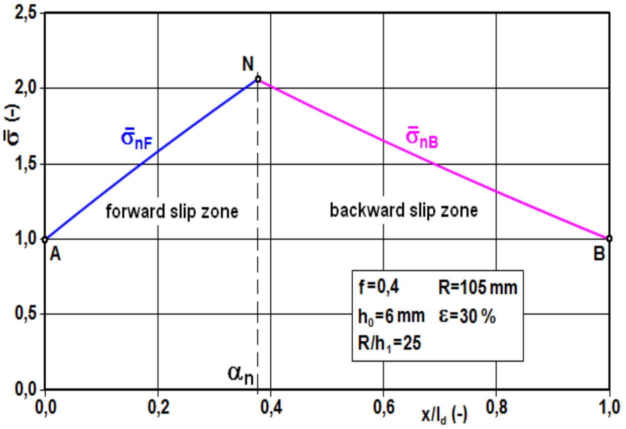

Fig. 3 The distribution of the relative normal contact stress in rolling gap at constant shear stress

3 New solution of differential equation

The following part will be represented the new analytical solution of eq.(1) with condition constant shear stress. The analytical solution of the eq.(1) is based on description of circular arch by equation of the circle as is showing in **Fig. 4**.

DOI 10.12776/ams.v21i1.549
p-ISSN 1335-1532
e-ISSN 1338-1156

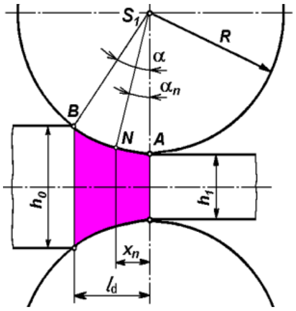

Fig. 4 The description of contact arc by the circle

The new form of eq.(7) after modification will be:

$$d\overline{\sigma}_n - \frac{dy}{y} \pm f \cdot \frac{dx}{y} = 0 \tag{23.}$$

The solution of eq.(23) can be obtained by its integration [14-19]:

$$\int d\overline{\sigma}_n = \int \frac{dy}{y} \mp f \int \frac{dx}{y} \tag{24.}$$

While the left side of eq.(24) is simply integrated the right side consists from two integrals which can be described by the functions $F_1(x)$ a $F_2(x)$ and eq.(24) will take the following form:

$$\overline{\sigma}_n = C + F_1(x) \mp f \cdot F_2(x) \tag{25.}$$

where C is integration constant. The function dependence y=f(x) in differential eq. (23) is representing of the equation of the circle:

$$x^2 + (y - y_0)^2 = R^2 \ , \quad y_0 = R + \frac{h_1}{2} \tag{26., 27.}$$

where: - R – the roll radius
- h_1 – exit thickness of rolling material

If variable y is separated from eq.(26) and is carried out the differentiation of the eq.(28) then is obtained the following formula:

$$y = R + \frac{h_1}{2} - \sqrt{R^2 - x^2} \ , \quad dy = \frac{x}{\sqrt{R^2 - x^2}} \cdot dx \tag{28., 29.}$$

The determination of ratio dy/y from eq.(28), eq.(29) and substituting to the function $F_1(x)$ is obtained:

$$F_1(x) = \int \frac{dy}{y} = \int \frac{\frac{x}{\sqrt{R^2 - x^2}}}{R + \frac{h_1}{2} - \sqrt{R^2 - x^2}} dx = \int \frac{\frac{x}{R}}{\sqrt{1 - \left(\frac{x}{R}\right)^2}} \cdot \frac{dx}{R\left(m - \sqrt{1 - \left(\frac{x}{R}\right)^2}\right)} \tag{30.}$$

where *m* is a constant comprising the following parameters:

$$m = 1 + \frac{h_1}{2R} \tag{31.}$$

Simplification of eq.(30) can be obtained when a transformation from the rectangular coordinate system to polar coordinates is performed. From **Fig. 5** is resulting the determination of the polar coordinate φ of point $\mathrm{K}[\mathrm{x;y}]$.

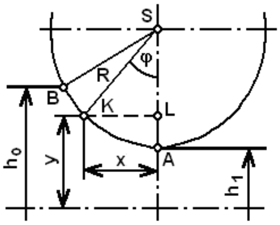

Fig. 5 Definition of the position of point K[x; y]

The transformation from Cartesian coordinates to polar with can be made as follows:

$$\frac{x}{R} = \sin\varphi\,, \quad \text{resp.} \quad dx = R\cos\varphi \cdot d\varphi \tag{32., 33.}$$

Substituting eq. (32) and eq.(33) to eq. (30) will obtained new formula which is labeled as function $F_1(m,\varphi)$ resulting from the transformation of Cartesian coordinates previously labeled as a function $F_1(x)$ into polar coordinates :

$$F_1(m,\varphi) = \int \frac{\sin\varphi}{\sqrt{1-\sin^2\varphi}} \cdot \frac{R\cos\varphi \cdot d\varphi}{R\left(m - \sqrt{1-\sin^2\varphi}\right)} = \int \frac{\sin\varphi}{m-\cos\varphi} \cdot d\varphi \tag{34.}$$

To calculate the integral eq.(34) will be applied following substitution:

$$t = m - \cos\varphi\,, \qquad dt = \sin\varphi \cdot d\varphi \tag{35., 36.}$$

$$F_1(m,\varphi) = \int \frac{\sin\varphi}{m-\cos\varphi} d\varphi = \int \frac{dt}{t} = \ln t = \ln(m - \cos\varphi) \tag{37.}$$

Graphical visualization of eq.(37) is shown in **Fig. 6**. Function $F_1(m,\varphi)$ throughout the project space shall take negative values.

The next step is definition of function $F_2(x)$ from eq.(25) as follows:

$$F_2(x) = \int \frac{dx}{y} = \int \frac{dx}{y_0 - \sqrt{R^2 - x^2}} = \int \frac{\frac{dx}{R}}{m - \sqrt{1 - \left(\frac{x}{R}\right)^2}} \tag{38.}$$

DOI 10.12776/ams.v21i1.549

p-ISSN 1335-1532
e-ISSN 1338-1156

and their transformation from the Cartesian coordinates ($F_2(x)$) into the polar coordinates ($F_2(m, \varphi)$):

$$F_2(m,\varphi) = \int \frac{\cos\varphi \cdot d\varphi}{m - \sqrt{1-\sin^2\varphi}} = \int \frac{\cos\varphi}{m-\cos\varphi} \cdot d\varphi . \tag{39.}$$

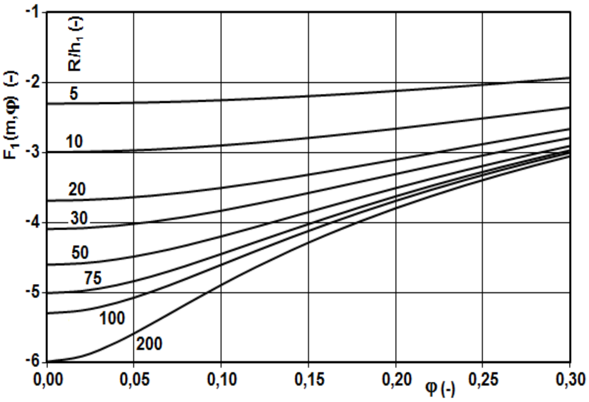

Fig. 6 Graphical visualization of function *F1(m, φ)*

The following substitutions are introduced for solution of the integral:

$$tg\frac{\varphi}{2} = z\ , \quad \cos\varphi = \frac{1-z^2}{1+z^2}\ , \qquad d\varphi = \frac{2dz}{1+z^2} \tag{40.}$$

Substituting eq. (40) to eq. (39) will obtained formula:

$$F_2(m,\varphi) = \frac{2}{m+1}\int \frac{1-z^2}{\left(1+z^2\right)\cdot\left(a+z^2\right)} \cdot dz\ , \tag{41.}$$

where: $a = \dfrac{m-1}{m+1}$ (42.)

Next procedure for solving the integral eq.(41) consists in its decomposition into partial fractions:

$$F_2(m,\varphi) = \frac{2}{m+1}\left[\frac{2}{a-1}\int\frac{dz}{1+z^2} + \frac{1+a}{1-a}\int\frac{dz}{a+z^2}\right]. \tag{43.}$$

and solution of eq.(43) will be as follows:

$$F_2(m,\varphi) = \frac{2}{m+1}\left[\frac{2}{a-1} arctg\ z + \frac{1+a}{1-a}\cdot\frac{1}{\sqrt{a}} arctg\frac{z}{\sqrt{a}}\right] \tag{44.}$$

Reversing the introduction of the constant *a* into the eq.(44) and the use of substitution from eq.(40) yields the formula:

$$F_2(m,\varphi) = -2arctg\ z + \frac{2m}{\sqrt{m^2-1}} arctg\left(\sqrt{\frac{m+1}{m-1}} \cdot z\right) \qquad (45.)$$

$$F_2(m,\varphi) = \frac{2m}{\sqrt{m^2-1}} arctg\left(\sqrt{\frac{m+1}{m-1}} \cdot tg\frac{\varphi}{2}\right) - \varphi \qquad (46.)$$

Graphical visualization of eq.(46) is shown in **Fig. 7**. Function $F_2(m,\varphi)$ throughout the project space shall take positive values.

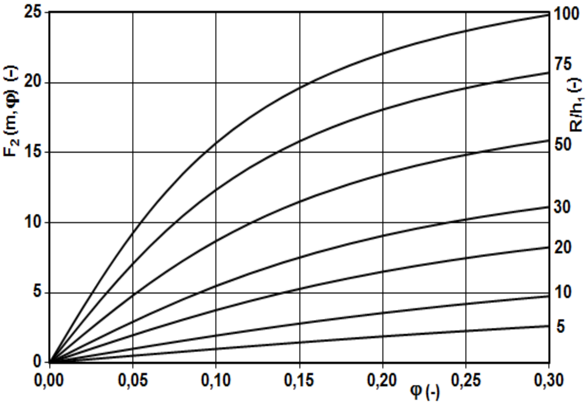

Fig. 7 Graphical visualization of function $F_2(m, \varphi)$

When the functions $F_1(m, \varphi)$ and $F_2(m, \varphi)$ are substituted into eq.(25) then receives the following formula:

$$\bar{\sigma}_n = C + F_1(m,\varphi) \mp f \cdot F_2(m,\varphi) \qquad (47.)$$

and will be obtained analytical solution for the relative normal contact stress in complex form:

$$\bar{\sigma}_n = C + \ln(m - \cos\varphi) \mp f \cdot \left(\frac{2m}{\sqrt{m^2-1}} arctg\left(\sqrt{\frac{m+1}{m-1}} \cdot tg\frac{\varphi}{2}\right) - \varphi\right) \qquad (48.)$$

where: C – is an integration constant

Equations describing the distribution of relative normal contact stress along rolling gap at constant shear stress will have the following forms:

- for backward slip zone:

$$\bar{\sigma}_{nB} = C_B + \ln(m - \cos\varphi) - f \cdot \left(\frac{2m}{\sqrt{m^2-1}} arctg\left(\sqrt{\frac{m+1}{m-1}} \cdot tg\frac{\varphi}{2}\right) - \varphi\right) \qquad (49.)$$

- for forward slip zone:

$$\bar{\sigma}_{nF} = C_F + \ln(m - \cos\varphi) + f \cdot \left(\frac{2m}{\sqrt{m^2-1}} arctg\left(\sqrt{\frac{m+1}{m-1}} \cdot tg\frac{\varphi}{2}\right) - \varphi\right) \qquad (50.)$$

DOI 10.12776/ams.v21i1.549 p-ISSN 1335-1532
e-ISSN 1338-1156

Rolling process is realized without forward and backward stretching forces, without material hardening during plastic deformation and without roll flattening. According to **Fig. 1** polar coordinate for point **B** is $\varphi=\alpha$ and for point **A** is $\varphi=0$. In these points horizontal stress is $\sigma_x=0$. In order to perform a condition of plasticity for these points must be valid $\sigma_n=\sigma_a$. The integration constant C_B for the backward slip zone shell be determined from the condition $\bar{\sigma}_{nB}=1$ a $\varphi=\alpha$ and eq.(49) as follows:

$$C_B = 1-\ln(m-\cos\alpha)+f\cdot\left(\frac{2m}{\sqrt{m^2-1}}arctg\left(\sqrt{\frac{m+1}{m-1}}\cdot tg\frac{\alpha}{2}\right)-\alpha\right) \tag{51.}$$

Also integration constant C_F for the forward slip zone shell be determined from the condition $\bar{\sigma}_{nF}=1$ a $\varphi=0$ and eq.(50) as follows:

$$C_F = 1-\ln(m-1) \tag{52.}$$

Substituting equations describing of the integration constants eq.(51) and eq.(52) into eq.(49) and eq.(50) will obtained the final analytical solutions for calculation of relative normal contact stress in backward and forward slip zones:

$$\bar{\sigma}_{nB} = 1+f\cdot\left(\frac{2m}{\sqrt{m^2-1}}arctg\left(\sqrt{\frac{m+1}{m-1}}\cdot tg\frac{\alpha}{2}\right)-\alpha\right)+\ln\left(\frac{m-\cos\varphi}{m-\cos\alpha}\right)-f\cdot\left(\frac{2m}{\sqrt{m^2-1}}arctg\left(\sqrt{\frac{m+1}{m-1}}\cdot tg\frac{\varphi}{2}\right)-\varphi\right) \tag{53.}$$

$$\bar{\sigma}_{nF} = 1+\ln\left(\frac{m-\cos\varphi}{m-1}\right)+f\cdot\left(\frac{2m}{\sqrt{m^2-1}}arctg\left(\sqrt{\frac{m+1}{m-1}}\cdot tg\frac{\varphi}{2}\right)-\varphi\right) \tag{54.}$$

The eq.(53) and eq.(54) can be written in short form as sum of two functions:

$$\sigma_n \approx F_1(m,\varphi)\mp f\cdot F_2(m,\varphi) \tag{55.}$$

where: $F_1(m,\varphi)$ – is depend only on geometric constant m and polar coordinate φ
$F_2(m,\varphi)$ – is depend on geometric constant m , polar coordinate φ and friction coefficient f

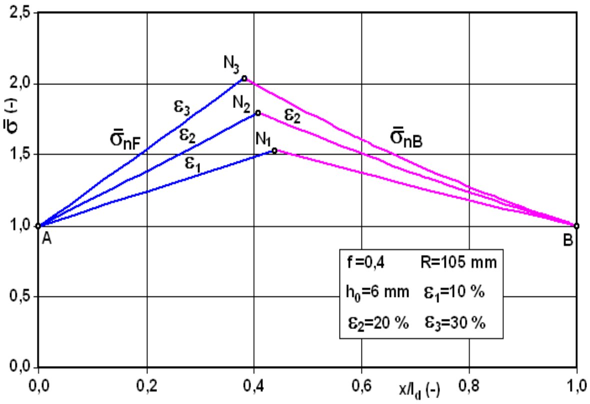

Fig. 8 The distribution of the relative normal contact stress in rolling gap with constant shear stress

DOI 10.12776/ams.v21i1.549
p-ISSN 1335-1532
e-ISSN 1338-1156

Geometric visualization of eq.(53) and eq.(54) is presented in **Fig. 8**. The curves represent the development of the relative normal contact stress in rolling gap in the condition of constant shear stress in dependence to the relative coordinate x/l_d.
The parameters are the relative thickness deformation and friction coefficient (f=0,4). Maximal value of the relative normal contact stress in the rolling gap is observed at neutral point. Increasing of relative deformation of thickness has resulted in the growth of relative normal contact stress and shifting of neutral point in direction to point **A** i.e. towards to the exit plane of rolling gap. The presented solution is valid to the rolled material which does not working hardening during his processing.

4 Conclusion

The analytical solution differential equation describing stress state in the rolling gap with condition constant shear stress is given in this paper. The first analytical solution of the differential eq.(1) was mentioned by the author Gubkin using a simplified describtion of the circular arch of the contact zone by the straight line and later by the parabola. A constant shear stress was used as further simplify the author. However these simplifications have impact to precision of the calculation of distribution of the normal stress in the rolling gap. The contribution of this paper is the new description of the circular arch of the contact zone by the equation of a circle. Approximation of the circular arch by the equation of a circle causes a problem to obtain analytical solution of differential eq.(1). A new analytical approach for solution of this case is based on the transformation from Cartesian coordinates (σ–x) to polar coordinates (σ–φ). Description of distribution of the relative normal contact stress on rolls is represented by the sum of two independent functions $\overline{\sigma}_n \approx F_1 + f \cdot F_2$. The first function has a logarithmic form $F_1 = F_1(m, \varphi)$ and describes the geometry of the rolling gap and second function $F_2 = F_2(m, \varphi)$ describes shear stress between the rolls and the rolling material. The second function has character of the wrapping angle and is independent on friction coefficient. The new approach for solution of eq.(1) allows too obtained calculation for case when shear stress is not a constant.

Acknowledgments
This work was financially supported by the VEGA 1/0325/14 project and the project "Technological preparation of electrotechnical steels with high permeability for electrodrives with higher efficiency" which is supported by the Operational Program "Research and Development" ITMS 26220220037, financed through European Regional Development Fund.

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Nomenclature

σ_n – normal contact stress [MPa]
$\overline{\sigma}_n$ – relative normal contact stress [–]
$\overline{\sigma}_{nB}$ – relative normal contact stress (Backward slip zone) [–]
$\overline{\sigma}_{nF}$ – relative normal contact stress (Forward slip zone) [–]
$\overline{\sigma}$ – sigma function (average relative normal contact stress) [–]
τ – shear stress [MPa]
σ_x , σ_y – principal stress (σ_3, σ_1) [MPa]
σ_a – actual resistance to deformation [MPa]
x , y – rectangular coordinates [m]
R , φ – polar coordinates [m, rad]
dx , dy – coordinate differentials x and y [–]
m – constant differential equation [–]
α – gripping angle [rad]
α_n – neutral angle [rad]
l_d – length of contact arc [m]
h_0 , h_1 – thickness before and after deformation [m]
h_n – thickness in neutral section [m]
h_{av} – average thickness [m]
Δh – absolute reduction [m]
ε – relative reduction [–]
f – friction coefficient [–]
R – radius of rollers [m]
C_B – integration constant (Backward slip zone) [–]
C_F – integration constant (Forward slip zone) [–]

DOI 10.12776/ams.v21i1.549 p-ISSN 1335-1532
e-ISSN 1338-1156