MODELING AND VIBRATION ANALYSIS OF COMPLEX ANNULAR SYSTEM WITH USING CYCLIC SYMMETRY FEATURE

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Abstract
In the paper the free transverse vibration of representative toothed gear is considered. Analysed system is treated as an annular plate with complex geometry. The discussion is focused on using the cyclic symmetry modelling technique to vibration analysis of the system under the study. The method of creation of the toothed gear simplified FE model is proposed. The problems related to the determination of critical rotational speeds of the considered system are discussed. Campbell diagram for the studied system is elaborated and the method of designation of the safe working zone of the wheel is presented. Experimental investigations conducted in order to evaluate the proposed method of elaboration of FE models are presented. It is important to note that the issues presented in this work have a practical value for design engineers dealing with the dynamics of rotating systems.

Keywords: toothed gear, transverse vibration, Campbell diagram, cyclic symmetry modeling, finite element modeling (FEM), dynamic mechanical analysis

1 Introduction

Problems of transverse vibration of circular-symmetry plate systems have been the subject of many investigations [1-16]. This is due to the fact that some rotating systems can be treated as circular-symmetry plates, in which both their shape and dimensions are covered by design of these systems. In papers [1, 3, 9-11, 17, 18] authors analyse free vibrations of toothed gears by using finite element (FE) technique. In papers [2, 4] the FE technique is utilized to the solving process of the transverse vibration of circular-symmetry plates with complex geometry and physical properties. The authors of papers [5, 6, 9, 14, 16] included the cyclic symmetry modelling in the analysis of vibration problems of some compound systems (toothed gears and others). Problems of critical rotational speeds and safe working zone of rotating systems are considered by authors of papers [1, 3, 8]. In the articles [7, 8, 12] the problem of transverse vibration of circular plates with holes is solved by using modified boundary element method. Monographs [13, 15, 19] present the fundamental vibration theory of plate systems. The work [20] deals with the model updating technique of FE models. In above paper the free transverse vibration of a toothed gear found in a high-speed gearbox is analysed using finite element technique. Considered system is treated as a compound annular plate installed on the hollow

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stepped shaft. The principal objective is to elaborate the method of creation of simplified FE model of the wheel which will have satisfactory dynamic properties. Then the problems related to the determination of the safe working zone of the wheel are discussed. This paper continues recent author's investigations concerning the transverse vibration of compound plate systems [21].

2 Formulation of the problem

Discussed system is a toothed gear which has the geometry as it is displayed in **Fig. 1**. Primary parameters characterizing the system (among others geometrical dimensions defined as shown in **Fig. 2** and **3**) are presented in **Table 1**. As mentioned earlier the considered system is composed of the annular plate with geared rim installed on the hollow shaft. The disc of the wheel contains discontinuities in a form of holes distributed over predetermined diameter (see **Fig. 1**). In the proposed simplified models the teeth are omitted and the toothed ring is simplified down to the regular ring (see **Figs. 2** and **3**). It allows utilizing in the modelling process the cyclic symmetry feature of the system which is occurred after simplification of the toothed rim (see **Fig. 3**). In this work during the vibration problem analysis the centrifugal effect due to rotation of the system is included. In accordance with the circular symmetry plate vibration theory [13-15], the particular natural frequencies of vibration are denoted as ω_{mn}, where m refers to the number of nodal circles and n is the number of nodal diameters.

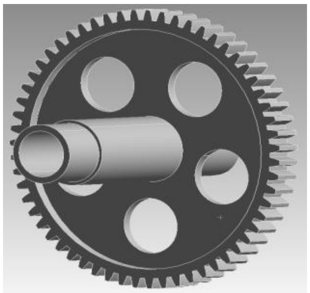

Fig. 1 Considered gear

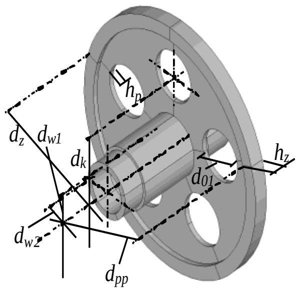

Fig. 2 Simplified model

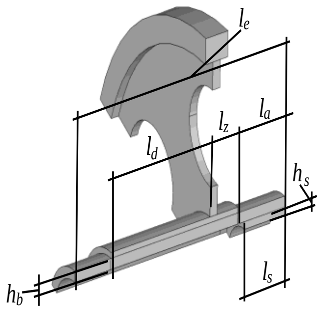

Fig. 3 The cyclic symmetry sector of the model

Table 1 Parameters characterizing the system

d_z [mm]		d_{pp} [mm]	d_k [mm]	d_{w1} [mm]	d_{w2} [mm]	d_{01} [mm]	h_z [mm]	h_p [mm]	h_b [mm]
110	105.4	98	62.7	26.7	18.8	22.4	7	2.3	2.6
h_s [mm]	d_p [mm]	l_a [mm]	l_e [mm]	l_d [mm]	l_s [mm]	l_z [mm]	ν	E [MPa]	ρ [kg/m^3]
1.8	110	14.9	66.7	31.4	12.7	8.9	0.3	$2.08 \cdot 10^5$	$7.83 \cdot 10^3$

2.1 Finite element representations

In this section the finite element models of the system under study are formulated. Three simplified FE models are discussed. The geometrical dimensions of the simplified models are taken from the gear geometry (see **Fig. 2**). In the first two FE models the entire geometry of the

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system is used. The difference refers to the outer diameter of the regular ring of the model. In the first model, the outer diameter of the rim is equal to the pitch diameter of the gear. Second FE model has the rim outer diameter equal the feet diameter of the gear (see **Table 1**). In the third FE model the cyclic symmetry of the system is utilized. This model is one-fifth part of the first FE model (see **Fig. 3**). To be in accordance with the reference solution, for each simplified model, some technical data of the model rim are selected experimentally. In the first and third model it is assumed that with the exception of Young's modulus (E_{01}), the technical data of the rim are the same as for the gear. In the second model case, both the mass density (ρ_{02}) and Young's modulus (E_{02}) of the rim are selected during calculations. The error between the precise and the simplified FE models is defined by [20]:

$$\varepsilon = \left(\omega^f - \omega^e\right) / \omega^e \times 100[\%] \quad (1.)$$

where: ε - frequency error

ω^f - natural frequency of the simplified FE model

ω^e - natural frequency of the reference FE model

In order to obtain reference values for the natural frequencies, for considered gear, a high resolution FE model (reference FE model) is set up, which contains all essential construction details of the real system. Each of elaborated models is meshed by using standard procedures of the ANSYS program. A 3-D solid mesh is prepared and the 10-node tetrahedral element (solid187) with three degrees of freedom in each node is employed to build each FE model (reference and simplified). The reference FE model is displayed in **Figs. 4** and **5**. This model includes 93423 solid elements. Proposed simplified FE models are shown in **Figs. 6, 7** and **8**. The first simplified FE model comprises 19725 solid elements. The second simplified FE model includes 17067 solid elements. However the third simplified FE model (cyclic symmetry model) consists of only 3984 solid elements. For all models presented here, calculations were conducted for a wide range of frequencies until the natural frequency ω_{18} was determined. The centrifugal effect is accounted for by determining the stress distributions resulting from rotation for each FE model during the static analysis computational step (pre-stress effect). This stress distribution is included in the computational step associated with the modal analysis [1, 14].

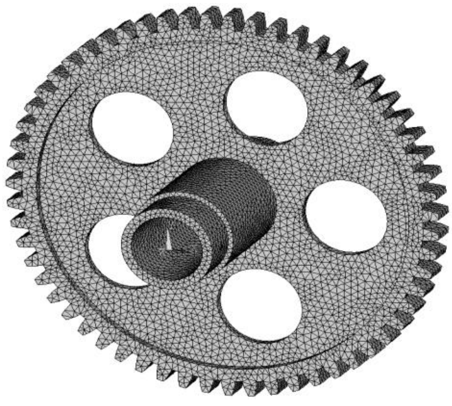

Fig. 4 Reference FE model of the gear (front view)

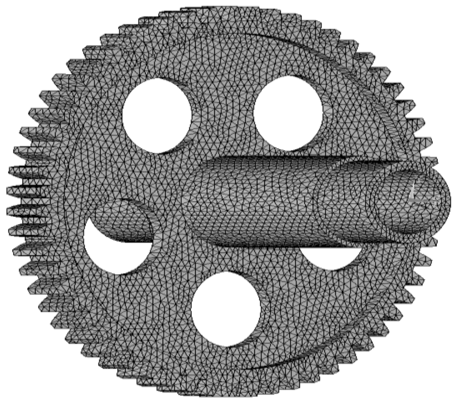

Fig. 5 Reference FE model of the gear (back view)

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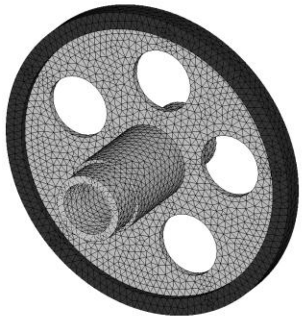

Fig. 6 First simplified FE model (19725 elements)

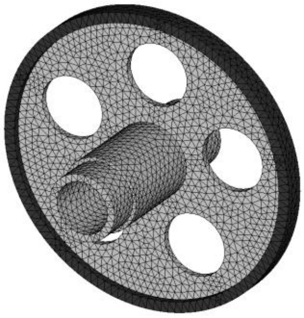

Fig. 7 Second simplified FE model (17067 elements)

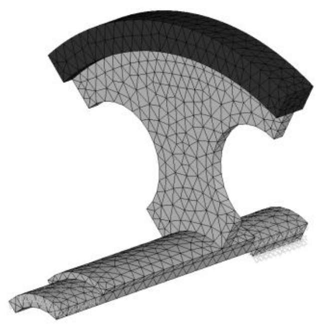

Fig. 8 Third simplified FE model (3984 elements)

2.2 Numerical analysis

Numerical analysis results of the considered system of free transverse vibration are obtained using FE models. The calculations were conducted in two stages. In the first calculation phase it is assumed that the considered system and simplified models rotate at angular velocity θ_0 = 80 [rad/s]. **Table 2** shows the values of E_{01}, E_{02} and ρ_{02}, for which satisfactory results were achieved for the simplified models. In **Table 3** values of the natural frequencies of the reference model are displayed. Due to space limitation, displaying the results related to simplified FE models have been limited to show the frequency error (1) of the third simplified model (see **Table 4**). They are the best achieved results. The second calculation phase involves the verification of the proposed simplified models with taking into account the centrifugal effect. In this case the calculation is conducted assuming that discussed models are rotated at angular velocity θ_0 = 1047 [rad/s] which corresponds to high velocity of centrifugation [1, 14]. As would be expected the best verification results are achieved for the third simplified FE model of the toothed gear. Appropriate results (the frequency error) are shown in **Table 5**. Achieved verification results are satisfactory. Analyzing the received results it is noted that for frequencies of ω_{15} and ω_{25} there is separation of their values. Each of the mentioned frequencies has two different values and each value corresponds to a different shape of natural form. It is caused by the five through-holes in the disk (see **Fig. 1**) and is consistent with the results reported in the literature [12, 14].

Table 2 Parameters of rim of simplified FE models

No.	E_{01} [MPa]	E_{02} [MPa]	ρ_{02} [kg/m^3]
1	$1.3\cdot10^5$	-	-
2	-	$2.2048\cdot10^5$	$1.4094\cdot10^4$

Table 3 Values of the natural frequencies ω_{mn} [Hz] (θ_0 = 80 [rad/s], reference FE model)

		n								
		0	1	2	3	4	5	6	7	8
m	1	794	603	1260	3311	6165	9808 9938	14210	18927	23970
	2	6191	6811	7955	12010	13739	16499 23754	23641		
	3	15061	16986	18109	22181					

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Table 4 Values of the frequency errors ε_{mn} [%] (θ_0 = 80 [rad/s], third simplified FE model)

		N								
		0	1	2	3	4	5	6	7	8
m	1	-1.64	0	1.42	0.18	0.06	-0.16 0.36	-1.76	2.56	2.6
	2	1.31	2.1	3.42	5.03	6.44	4.41 5.03	6.35		
	3	3.09	3.36	2.24	3.34					

Table 5 Values of the frequency errors ε_{mn} [%] (θ_0 = 1047 [rad/s], third simplified FE model)

		n								
		0	1	2	3	4	5	6	7	8
m	1	-1.49	0.16	1.4	0.21	-0.08	-0.15 -0.37	-1.74	2.6	2.61
	2	1.32	2.08	3.43	5.03	6.43	4.41 5.04	6.3		
	3	3.08	3.34	2.22	3.33					

The further step of calculations was referring to the issues concerning the designation of critical rotational speeds of the wheel and determination of the so-called safe work zones. In this work the notion of the critical rotational speed is referred to the speed at which can lead to the excitation of one of the reported natural frequencies of the considered system [1, 3]. Dynamic analysis is preferably performed with using a so-called Campbell diagram. This is a convenient method of displaying the relationship between toothed gear natural frequencies and transmission system excitations [3, 14]. To develop the diagram the calculations are performed with assumption that the discussed system rotates at angular velocity which values are given in **Table 6**. During gear operation process the most significant load that can cause vibration is a periodically varying load on the rim which comes from the meshing with another wheels [1, 3]. The forced frequency due to the meshing is determined by the following relation [1, 14]

$$f_k = k(n_1 z/60) \tag{2.}$$

where: $k = 1,2$

$n_1 [rpm]$ - rotational speed

z - number of teeth

Table 6 Values of the angular velocity of the system

angular velocity θ_0 [rad/s]							
0	160	320	400	560	720	880	1047

For discussed system, the gear teeth number is z = 59. In the **Fig. 9** the Campbell diagram associated with the discussed wheel is shown. On this diagram, excited frequencies are plotted on a vertical axis, and the gear rotational speed is plotted on the horizontal axis (see **Fig. 9**). Natural frequencies achieved from numerical solution are plotted as near-horizontal curves. The zone between the vertical dashed lines is the so-called work zone of the wheel. Vertical dashed lines correspond to the minimum and maximum values of the gear rotational speed. Vibration resonance phenomenon occurs when forcing frequency (Eq. 2) matches as to its value with some

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natural frequency of the wheel. Particularly it is important the resonance which comes from the forcing frequency f_1 (see Eq. (2)). Points where the near-horizontal ω_{mn} curves intersect the straight line (2) indicate the speed at which the resonance will be excited (see **Fig. 9**). For example, the rated excitation speed for the natural frequency of ω_{13} is 3389 [rpm] (see **Fig. 9**). On the diagram there are marked the points which appear from the intersection of the curves ω_{mn} by line f_1. Taking into account achieved results it is seen that in the working wheel zone the frequencies of ω_{14}, ω_{20}, ω_{21} and ω_{22} may be excited by the forcing frequency f_1 while the frequencies of ω_{15}, ω_{23}, ω_{24}, ω_{16}, ω_{30}, ω_{25}, ω_{31} and ω_{32} may be excited by the forcing frequency f_2. Values of the critical rotational speeds at which the resonance phenomenon may occur are shown in Table 7. It is visible that in the working gear zone the resonance phenomenon may occur. Due to the relatively high amplitudes, particularly dangerous is the resonance which comes from the forcing frequency f_1. To avoid the resonance problem, the changes of the system design or the working system zone should be considered.

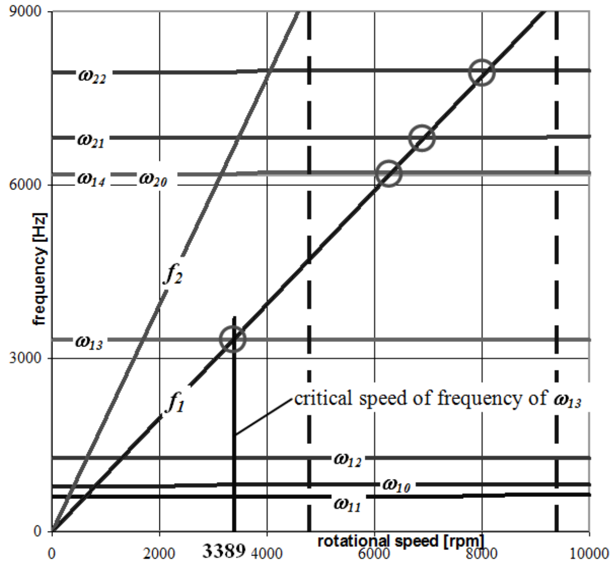

Fig. 9 Campbell diagram

Table 7 Values of the critical rotational speeds of the studied system

	n_1 [rpm]												
	4990	5050	6110	6275	6301	6940	6995	7230	7665	8105	8400	8645	9225
f_1	–	–	–	ω_{14}	ω_{20}	ω_{21}	–	–	–	ω_{22}	–	–	–
f_2	ω_{15}	ω_{15}	ω_{23}	–	–	–	ω_{24}	ω_{16}	ω_{30}	–	ω_{25}	ω_{31}	ω_{32}

3 Discussion

In this section the experimental investigations conducted in order to evaluate the proposed method of elaboration of FE models are presented. To some extend discussed in this section investigations were reported in the paper [9]. Geometry of the object used in the experiment is displayed in **Figs. 10-12**. Parameters of the object are shown in **Table 8**. LMS measurement

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environment is used in the experimental study. The measuring set contained the PCB model 086C03 type modal hammer, accelerometer PCB model 353B18, LMS SCADA data acquisition system, and SCM-V4E type measuring module supported by LMS Test.Lab software. Experimental study is conducted to identify natural frequencies and corresponding normal modes related to the transverse vibration of the discussed object. Thirty two measurement points were marked on the rim of tested object (see **Fig. 10**). The accelerometer was placed at each marked point to measure the system response in the transverse direction. Tested object was excited by the modal hammer.

Table 8 Parameters characterizing the object

d_z [m]	d_w [m]	d_l [m]	l_w [m]	l_r [m]	E [Pa]	ρ [kg/m^3]	ν
0.191	0.159	0.02	0.002	0.008	$2.1\cdot10^{11}$	$7.85\cdot10^{3}$	0.28

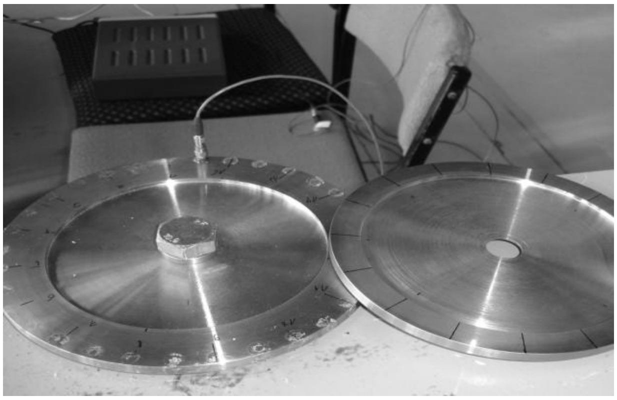

Fig. 10 Measuring test

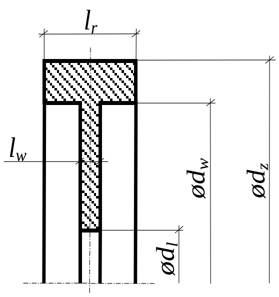

Fig. 11 Geometrical model of the object (dimensions)

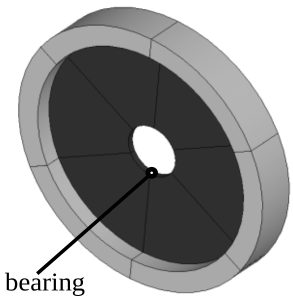

Fig. 12 Geometrical model (3-D shape)

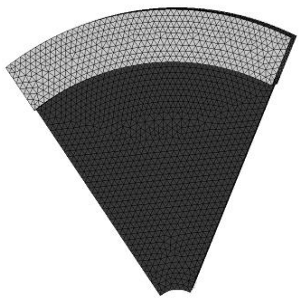

Fig. 13 FE model of the object

Values of the excited and measured natural frequencies are shown in **Table 9**. For the discussed object the FE model is elaborated with utilization the cyclic symmetry feature of the system (see **Fig. 13**). The elaborated FE model is one-sixth part of the full system. During the meshing process the same procedures as for the third simplified FE model (see Section 3) were used. Values of the frequency error related to the FE model are listed in the **Table 10**. Analyzing the obtained results it is noticeable that only for two frequency cases (frequency of ω_{10} and ω_{12}) the

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frequency error is above 10 %. So the obtained results can be considered as satisfactory and the proposed method of elaboration of FE models as a proper.

Table 9 Values of the natural frequencies ω_{mn} [Hz] (measuring test)

		n						
		0	1	2	3	4	5	6
m	1	263.8	141.9	576.6	1697	3272	5233	7463
	2	1847	2247	2948	3976	5318	6941	
	3	4397	5001	6453				

Table 10 Values of the frequency errors ε_{mn} [%]

		n						
		0	1	2	3	4	5	6
m	1	-10.39	5.29	11.81	4.3	2.08	1.13	0.8
	2	4.76	-0.31	-1.7	-1.16	0.17	0.98	
	3	3.02	3.3	1.95				

Conclusions

Present paper deals with free transverse vibration of the selected toothed gear which is treated as an annular plate with complex geometry. Three simplified FE models of the wheel are discussed. The most attractive is the model where the cyclic symmetry feature of the system is utilized. Reported in the literature the phenomenon of separation of values of some frequencies is confirmed. Discussion related to determination of critical rotational speed and safe work zone of the considered system is conducted. Obtained results in the experimental investigation confirm the attractiveness of the proposed method of elaboration of simplified FE models. It is important to note that because of using the commercial software (ANSYS program), presented investigation can be attractive for design engineers dealing with the dynamics of rotating systems.

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