

## SIMPLE METHODOLOGY FOR CALCULATING THE CONSTANTS OF GAROFALO EQUATION

Rudolf Pernis<sup>1)\*</sup>

<sup>1)</sup> Alexander Dubcek University of Trencin in Trencin, Faculty of Special Technology, Slovak Republic

Received 08.08.2017

Accepted 11.10.2017

\*Corresponding author: e-mail: rudolf.pernis@tuni.sk, Tel.: +421327400274, Pri parku 19, 911 06 Trenčín, Slovak Republic

### Abstract

The examination of the high temperature plastic properties of metallic materials was realised by the torsion plastometer SETARAM or by the compression plastometer DIL805A/D. The brass CuZn30 was used as the test material. Peak stress detection was performed for two independent variables, temperature and velocity of deformation. The set experimental plan forms the test array 5×4, i.e. five temperatures 650, 700, 750, 800, 850 °C and four strain rates of 0.5, 2.5, 12.5 and 25 s<sup>-1</sup>. It was necessary to evaluate measured data and to determine the mathematical model of the peak stress. For this aim, the Garofalo equation was used. This equation contains 4 material constants. The currently method used to determine the material constants takes a relatively long time and requires a number of auxiliary calculations. In this method, nonlinear regression is often used, which requires the initial estimation of parameters. The article presents a mathematical analysis and a simple methodology for calculating the material constants of the Garofalo equation. A general linear regression is used for the calculation that does not require an initial estimate of the material constants. The numerical calculation of material constants is documented on the measured peak stress data.

**Keywords:** Garofalo equation, hot working, brass CuZn30, torsion, linear regression

### 1 Introduction

The Garofalo empirical equation [1, 2] is used to describe the high temperature deformation, which describes the strain rate in relation to the flow stress and the absolute temperature

$$\dot{\varphi} = C \cdot [\sinh(\alpha \cdot \sigma_p)]^n \cdot \exp\left(-\frac{Q}{RT}\right) \quad (1.)$$

kde  $\dot{\varphi}$  [s<sup>-1</sup>] – strain rate  
 T [K] – absolute temperature  
 $\sigma_p$  [MPa] – flow stress  
 Q [J.mol<sup>-1</sup>] – activation energy of deformation  
 R [J.K<sup>-1</sup>.mol<sup>-1</sup>] – universal gas constant  
 n [–] – material constant  
 C [s<sup>-1</sup>] – material constant  
 $\alpha$  [MPa<sup>-1</sup>] – material constant

Extensive analysis of the Garofalo equation (1) and the flow of material has been carried out by several authors [3, 4, 5]. Material constants are determined from the measured actual stress values, which are defined by the temperature and the strain rate. [6, 7]. Different mathematical methods and nonlinear regression [11, 12] are used for searching constants  $n$ ,  $Q$  a  $C$  [8, 9, 10].

*Calculation of material constant  $\alpha$ .* The Eq. (1) allows a description of strain stress dependencies for specific thermodynamic conditions of forming. For high strain temperatures and low strain rates, the condition  $\alpha \cdot \sigma < 1$  applies, (the data in row of temperature 850 °C in **Table 4**), therefore Eq. (1) can be reduced to the form [13]

$$\dot{\varphi}' = C' \cdot \exp\left(-\frac{Q}{RT}\right) \cdot \sigma^{n_1} \quad (2.)$$

For condition  $\alpha \cdot \sigma > 1$ , (the data in row of temperature 650 °C in **Table 4**), that is fulfilled when forming at lower temperatures and higher strain rates, it is possible to reduce Eq. (1) to the form [13]

$$\dot{\varphi}' = C'' \cdot \exp\left(-\frac{Q}{RT}\right) \cdot \exp(\beta \cdot \sigma) \quad (3.)$$

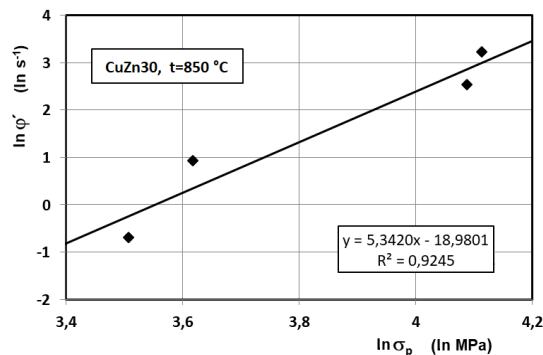
while

$$\beta = \alpha \cdot n_1 \quad (4.)$$

The following procedure is used in [3] to determine the material constant  $\alpha$ . The constants  $n_1$  and  $\beta$  are calculated from Eq. (2) and Eq. (3). For this aim, these equations have been transformed to the form of a line equation by using the logarithm

$$\ln \dot{\varphi}' = \ln C' - \frac{Q}{RT} + n_1 \cdot \ln \sigma \quad (5.)$$

$$\ln \dot{\varphi}' = \ln C'' - \frac{Q}{RT} + \beta \cdot \sigma \quad (6.)$$

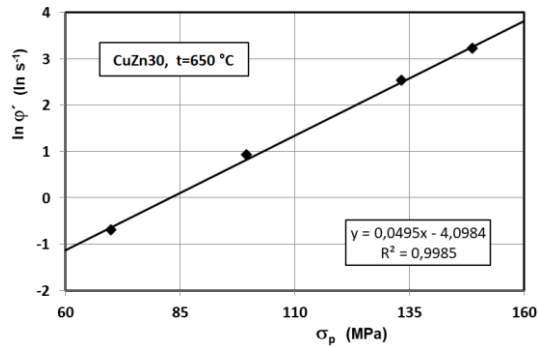


**Fig. 1** The line directive determines the constant  $n_1 = 5,3420$

The data in the row of temperature 850 °C in **Table 4** were used to calculate the constant  $n_1$  of Eq. (5). Similarly, the data in row of temperature 650 °C in **Table 4** were used to calculate the

constant  $\beta$  of Eq. (6) [13, 14]. Because the data at constant temperature were always used, the first two members on the right of equations (5) and (6) are also the constants. Eq. (5) expresses the dependence  $\ln \phi' - \ln \sigma$  and Eq. (6) the dependence  $\ln \phi' - \sigma$ . The graphical representation of these dependencies is in **Fig. 1** and **Fig. 2**. Obtained values  $n_i$  and  $\beta$  allow calculating the material constant  $\alpha$  for CuZn30 brass using the Eq. (4)

$$\alpha = \frac{\beta}{n_1} = \frac{0.0495}{5.342} = 0.00927 \text{ MPa}^{-1} \quad (7.)$$



**Fig. 2** The line directive determines the constant  $\beta = 0,0495$

*Calculation of material constant n.* If we use the logarithm operation on the Eq. (1), we get linearized form of this equation

$$\ln \phi' = \ln C - \frac{Q}{RT} + n \cdot \ln \sinh(\alpha \cdot \sigma_p) \quad (8.)$$

In Eq. (8) we use transformations  $y = \ln \phi'$  and  $x = \ln \sinh(\alpha \cdot \sigma_p)$ . At the same temperature  $T$ , the expression  $y_0 = \ln C - Q/RT$  is constant. Then the form of Eq. (8) will be

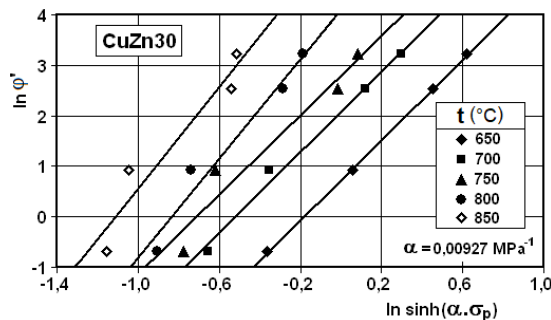
$$y = y_0 + n \cdot x \Big|_{T=\text{const}} \quad (9.)$$

where material constant  $n$  means gradient of the line (9). For constant temperatures Eq. (9) represents a set of lines. The graph of the lines will be view in the coordinates  $\ln \phi' - \ln \sinh(\alpha \cdot \sigma_p)$  for points with the same temperature [15]. **Fig. 3** shows five lines for our five temperatures. The gradients of these lines represent the material constant  $n$  for a corresponding temperature. The values of this material constant  $n$  are calculated in **Table 1** for every line. The resulting value of the material constant  $n$  for the whole test array is given by the arithmetic mean

$$n = \frac{1}{m} \cdot \sum_{i=1}^{m=5} n_i = \frac{21.8396}{5} = 4.3679 \quad (-)$$

**Table 1** Line parameters (constant  $n$ ) shown on the **Fig. 3**

i	$t_i$ (°C)	$n_i$ (-)	$R^2$
1	650	3.9724	0.9997
2	700	3.9798	0.9905
3	750	3.9515	0.9385
4	800	4.9117	0.9608
5	850	5.0242	0.9231
Sum	-	21.8396	-



**Fig. 3** Line plots at different temperatures determining material constant  $n$  [15]

*Calculation of material constant  $Q$ .* The material constant  $Q$  represents the activation energy of the deformation. To determine the value of this material constant we use Eq. (8) [16, 17]. This method is highly demanding for the data processing time and the determination of the material constant  $Q$  [18, 19, 20]. In Eq. (8) we introduce transformations  $y = \ln \sinh(\alpha \cdot \sigma_p)$  and  $x = 1000/T$ . Expression  $y_0 = (\ln \phi' - \ln C)/n$  means  $y$ -intercept of the line, because the strain rate is for the corresponding line constant. Eq. (8) takes a form [3, 15]

$$y = y_0 + k \cdot x \Big|_{\phi' = \text{const}} \quad (10.)$$

where

$$k = \frac{Q}{nR} \quad (11.)$$

The graph of the line will be shown in the coordinates  $\ln \sinh(\alpha \cdot \sigma_p) - 1000/T$  in points that have the same strain rate [15]. **Fig. 4** shows four lines for four used strain rates. The values of gradients  $k_i$  of every line are shown in **Table 2**. The resulting value of the constant  $k$  for the whole test array is given by the arithmetic mean

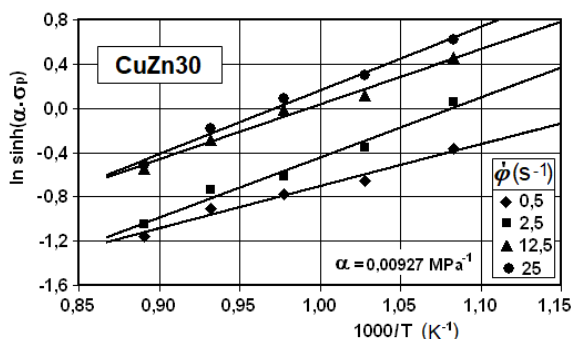
$$k = \frac{1}{m} \cdot \sum_{i=1}^{m=4} k_i = \frac{19.8648}{4} = 4.9662 \quad \text{K}$$

Based on Eq. (11) and after taking into account the factor  $1000/T$  [21], material constant  $Q$  can be calculated

$$Q = 1000 \cdot k \cdot n \cdot R = 1000 \cdot 4.9662 \cdot 4.3679 \cdot 8.314 = 180\,346 \quad \text{J} \cdot \text{mol}^{-1}$$

**Table 2** Line parameters (constant  $k$ ) shown on the **Fig. 4**

$i$	$\dot{\varphi}'_i \text{ (s}^{-1}\text{)}$	$k_i \text{ (K)}$	$R^2$
1	0.5	3.8066	0.9747
2	2.5	5.4007	0.9796
3	12.5	4.9652	0.9865
4	25	5.6923	0.9902
Sum	–	19.8648	–



**Fig. 4** Line plots at different strain rate determining material constant  $Q$  [15]

*Calculation of material constant  $C$ .* Material constant  $C$  is the last constant which appears in Garofalo equation. Eq. (1) can be written in the form

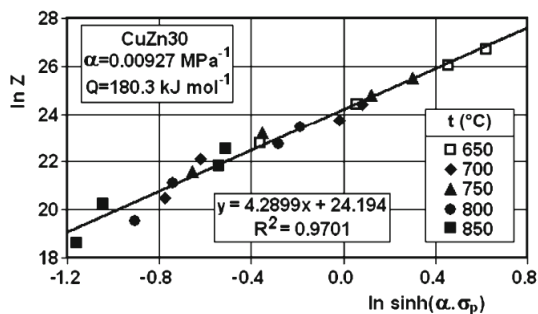
$$\varphi' \cdot \exp\left(\frac{Q}{RT}\right) = C \cdot [\sinh(\alpha \cdot \sigma_p)]^n \quad (12.)$$

The left side of the Eq. (12) represents Zener – Hollomon parameter  $Z$ :

$$Z = \varphi' \cdot \exp\left(\frac{Q}{RT}\right) \quad (13.)$$

From Eq. (12) and (13) is made next equation

$$\ln Z = \ln C + n \cdot \ln \sinh(\alpha \cdot \sigma_p) \quad (14.)$$



**Fig. 5** Determination of constant  $C$  by using Zener-Hollomon parameter [15]

Because the activation energy  $Q$  has already been calculated, it is possible to calculate Zener – Hollomon parameter from Eq. (13). In Eq. (14) we introduce transformations  $y = \ln Z$  and  $x = \ln \sinh(\alpha \cdot \sigma_p)$ . The expression  $y_0 = \ln C$  means natural logarithm of material constant  $C$ . Now Eq. (14) has the form

$$y = y_0 + n \cdot x \quad (15.)$$

where material constant  $C$  signifies  $y$ -intercept  $y_0$  of the line. Eq. (15) represents one line for whole set of measured values. The graph of the line will be shown in the coordinates  $\ln Z - \ln \sinh(\alpha \cdot \sigma_p)$  (**Fig. 5**), from which it results

$$\ln C = 24.194 \quad a \quad C = 3.216034 \cdot 10^{10} \text{ s}^{-1}$$

For calculating the constants of Garofalo equation are used different mathematical methods, most often nonlinear regression. The new solution consists in the use of general linear regression.

## 2 Experimental Material and Methods

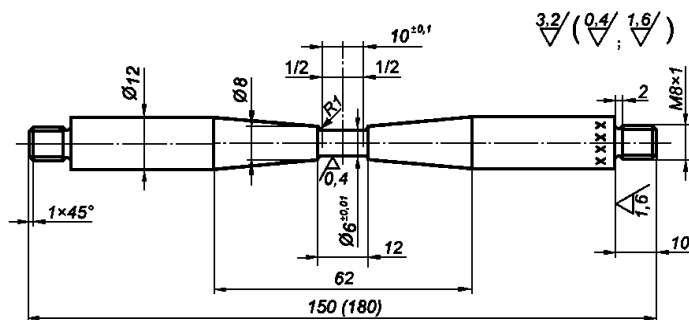
The CuZn30 brass was used for high temperature plastic deformation tests [15]. This brass belongs to the alpha brasses, which have good cold formability. It is the brass used in the munitions industry. The test material was in the form of bars with a diameter  $\phi$  12 mm. The chemical composition of this brass is given in **Table 3** and corresponds to the German standard DIN 17 660 w.n. 2.0265. Courses of stress in dependence on deformation were obtained on a torsion plastometer SETARAM. The dimensions of the test sample are given in **Fig. 6**. In order to use the Garofalo equation for the evaluation of the tests, it is necessary to follow the elaborated test plan. The test plan is represented by array  $5t \times 4\phi'$  (5 temperatures  $\times$  4 strain rates). Every element of the array signifies one measurement at defined temperature and deformation rate.

**Table 3** Chemical composition of CuZn30 brass (wt. %)

Element	Cu	Pb	Sn	As	Ni	Mn	Al	Si
Content	70.39	0.0004	0.0042	0.0001	0.0022	0.0003	0.0012	0.0002
Element	Fe	Sb	Bi	Cr	Cd	Ag	P	Zn
Content	0.023	0.003	0.0001	0.0001	0.0001	0.0001	0.0002	balance

$$\text{Test plan} = \begin{pmatrix} \sigma(t_1, \phi'_1) & \sigma(t_1, \phi'_2) & \sigma(t_1, \phi'_3) & \sigma(t_1, \phi'_4) \\ \sigma(t_2, \phi'_1) & \sigma(t_2, \phi'_2) & \sigma(t_2, \phi'_3) & \sigma(t_2, \phi'_4) \\ \sigma(t_3, \phi'_1) & \sigma(t_3, \phi'_2) & \sigma(t_3, \phi'_3) & \sigma(t_3, \phi'_4) \\ \sigma(t_4, \phi'_1) & \sigma(t_4, \phi'_2) & \sigma(t_4, \phi'_3) & \sigma(t_4, \phi'_4) \\ \sigma(t_5, \phi'_1) & \sigma(t_5, \phi'_2) & \sigma(t_5, \phi'_3) & \sigma(t_5, \phi'_4) \end{pmatrix}$$

Variables  $t_1$  to  $t_5$  represent temperatures 850, 800, 750, 700 and 650 °C. Variables  $\phi'_1$  to  $\phi'_4$  mean strain rates of 0.5, 2.5, 12.5 and 25  $\text{s}^{-1}$ . The matrix of tests represents  $5 \times 4 = 20$  measurements. Measured values of peak stress resulting from courses of stress in dependence on deformation in accordance with the test plan are shown in **Table 4**.



**Fig. 6** Test specimen for hot torsion tests

**Table 4** Peak stress of CuZn30 brass (MPa)

t (°C) \ φ' (s <sup>-1</sup> )	0.5	2.5	12.5	25
850	33.38	37.26	59.68	61.21
800	42.53	49.68	75.05	81.65
750	48.17	55.56	93.90	101.61
700	53.74	70.69	104.61	119.58
650	69.95	99.58	133.37	148.82

### 3 Results and Discussion

*New calculation methodology of material constant α.* In Eq. (5) and Eq. (6) the constants  $n_1$  and  $\beta$  mean the gradient of line. To calculate material constant  $\alpha$  it is not necessary to obtain complete regression equations according to **Fig. 1** a **Fig. 2**. It is sufficient to calculate only the gradients of lines that represent constants  $n_1$  a  $\beta$ . According to the theory of mathematical statistics, we define them as follows

$$\beta = \frac{s \cdot \sum_{i=1}^s \sigma_i \cdot \ln \varphi'_i - \sum_{i=1}^s \sigma_i \cdot \sum_{i=1}^s \ln \varphi'_i}{s \cdot \sum_{i=1}^s \sigma_i^2 - \left( \sum_{i=1}^s \sigma_i \right)^2} \Bigg|_{T_{\max}=\text{const}} \quad (16.)$$

$$n_1 = \frac{s \cdot \sum_{i=1}^s \ln \sigma_i \cdot \ln \varphi'_i - \sum_{i=1}^s \ln \sigma_i \cdot \sum_{i=1}^s \ln \varphi'_i}{s \cdot \sum_{i=1}^s \ln \sigma_i^2 - \left( \sum_{i=1}^s \ln \sigma_i \right)^2} \Bigg|_{T_{\min}=\text{const}} \quad (17.)$$

where  $s$  represents the number of strain rates at which the strain stress was measured (in **Table 4**  $s=4$ ). Numeric value of material constant  $\alpha$  is determined by Eq. (4), using values calculated from Eq. (16) and Eq. (17).

*New calculation methodology of material constant  $n$ ,  $Q$  a  $C$ .* Calculation procedure of material constants  $n$ ,  $Q$  and  $C$  is as follows, use logarithm on Eq. (1) and adjust it

$$\ln \sinh(\alpha \cdot \sigma_p) = -\frac{\ln C}{n} + \frac{1}{n} \cdot \ln \varphi' + \frac{Q}{nR} \cdot \frac{1}{T} \quad (18.)$$

Introduce next transformations in Eq. (18)

$$y = \ln \sinh(\alpha \cdot \sigma_p) \quad (19.)$$

$$f_1(\varphi') = \ln \varphi' \quad (20.)$$

$$f_2(T) = \frac{1}{T} \quad (21.)$$

and constants in Eq. (18) replace with substitutions

$$b_0 = -\frac{\ln C}{n} \quad (22.)$$

$$b_1 = \frac{1}{n} \quad (23.)$$

$$b_2 = \frac{Q}{nR} \quad (24.)$$

then Eq. (18) will take shape

$$y = b_0 + b_1 \cdot f_1(\varphi') + b_2 \cdot f_2(T) \quad (25.)$$

Unknown constants  $b_0$ ,  $b_1$  and  $b_2$  in Eq. (25) will be obtain by general linear regression, because functions  $f_1$  and  $f_2$  are known, defined by Eq. (20) and Eq. (21). The determinant equations have the following form

$$b_0 \cdot \sum_{i=1}^p 1 + b_1 \cdot \sum_{i=1}^p \ln \varphi'_i + b_2 \cdot \sum_{i=1}^p \frac{1}{T_i} = \sum_{i=1}^p y_i \quad (26.)$$

$$b_0 \cdot \sum_{i=1}^p \ln \varphi'_i + b_1 \cdot \sum_{i=1}^p \ln^2 \varphi'_i + b_2 \cdot \sum_{i=1}^p \frac{\ln \varphi'_i}{T_i} = \sum_{i=1}^p \ln \varphi'_i \cdot y_i \quad (27.)$$

$$b_0 \cdot \sum_{i=1}^p \frac{1}{T_i} + b_1 \cdot \sum_{i=1}^p \frac{\ln \varphi'_i}{T_i} + b_2 \cdot \sum_{i=1}^p \frac{1}{T_i^2} = \sum_{i=1}^p \frac{y_i}{T_i} \quad (28.)$$

where  $p$  represents total number of measurements (in **Table 4**  $p=20$ ). Eq. (26) to Eq. (28) are a system of three linear equations of three unknowns  $b_0$ ,  $b_1$  and  $b_2$ . After solving the system of equations, material constants will be calculated by Eq. (22) to Eq. (24)

$$C = \exp\left(-\frac{b_0}{b_1}\right) \quad (29.)$$

$$n = \frac{1}{b_1} \quad (30.)$$

$$Q = \frac{b_2}{b_1} \cdot R \quad (31.)$$

*Example of calculation of material constants:  $n$ ,  $Q$  and  $C$ .* For calculating the particular sums in Eq. (26) to Eq. (28) we use data from **Table 4**. The numerical form of these equations is as follows

$$20 \cdot b_0 + 29.838740 \cdot b_1 + 0.019642 \cdot b_2 = -6.521303 \quad (32.)$$

$$29.838740 \cdot b_0 + 90.302542 \cdot b_1 + 0.029304 \cdot b_2 = 0.502198 \quad (33.)$$



$$0.019642 \cdot b_0 + 0.029304 \cdot b_1 + 0.000019 \cdot b_2 = -0.005942 \quad (34.)$$

The system of three linear equations of three unknowns we recommend to solve using the determinants, through Cramer's rule

$$b_0 = -5.5367033$$

$$b_1 = 0.2234698$$

$$b_2 = 4966.2287$$

Material constants  $n$ ,  $Q$  and  $C$  are calculated from Eq. (29) to Eq. (31):

$$n = 4.4749$$

$$C = 5.7558 \cdot 10^{10} \text{ s}^{-1}$$

$$Q = 184764 \text{ J} \cdot \text{mol}^{-1}$$

Values of peak stress will be determined from the Garofalo equation (1) that should be adjusted for direct calculation of stress

$$\sigma_p = \frac{1}{\alpha} \operatorname{argsinh} \left\{ \left[ \frac{\phi'}{C} \cdot \exp \left( \frac{Q}{RT} \right) \right]^{1/n} \right\} \quad (35.)$$

Based on the specified material constants  $n$ ,  $C$ ,  $Q$ , and  $\alpha$ , the values of stress  $\sigma_p$  are calculated by Eq. (35). The rate of correlation between the measured and calculated values of peak stress gives the correlation coefficient ( $R^2 = 0.97998$ ). The comparison of the calculated material constants of the Garofalo equation, according to the old and new methodology is given in **Table 5**. The use of results obtained by plastometer in metal forming processes is documented by several publications [22, 23]. A special application in the rolling process is given in the publication [24].

#### 4 Conclusions

The results of determination of material constants  $\alpha$ ,  $n$ ,  $Q$  and  $C$  obtained by new methodology are in good agreement with the old methodology. The main benefit of the new method of calculating the material constants for the Garofalo equation lies in its simplicity and the speed of the calculation. Material constants are calculated directly as their final values

- The new methodology for calculating the constants of the Garofalo equation does not require an initial estimate of constants as nonlinear regression
- The calculation of material constants does not require complicated software as for nonlinear regression
- The new methodology does not require to calculate line equations, see on **Fig. 3** to **Fig. 5**
- The new methodology allows the calculation of constants in a fully automatic mode
- The Garofalo equation was linearized using logarithm what allows to calculate its constants through general linear regression
- The new methodology is suitable for the evaluation of the measured data by the torsion plastometer SETARAM and also by the compression plastometer DIL805A/D.

#### References

- [1] P. Garofalo: Transactions of the Metallurgical Society of AIME, Vol. 227, 1963, p. 351-355

- [2] C. M. Sellars, W. J. Mc Tegart: *International Metallurgical Review*, Vol. 17, 1972, p. 1-24, DOI: 10.1179/imt.1972.17.1.1
- [3] E.M. Mielnik: *Metalworking Science and Engineering*, McGraw-Hill, New York, NY, USA, 1991
- [4] S.H. Talbert, B. Avitzur: *Elementary Mechanics of Plastic Flow in Metal Forming*, John Wiley & Sons, New York, 1996
- [5] S. Spigarelli, M. Cabibbo, E. Evangelista, J. Bidulská: *Journal of Materials Science*, Vol. 38, 2003, No. 1, p. 81-88, DOI: 10.1023/A:1021161715742
- [6] J. Bidulská, I. Pokorný, T. Kvačkaj, R. Bidulský, M. Actis Grande: *High Temperature Materials and Processes*, Vol. 28, 2009, No. 5, p. 315-321, DOI: 10.1515/HTMP.2009.28.5.315
- [7] T. Kvačkaj, I. Pokorný: *Metalurgija*, Vol. 34, No. 4, 1995, p. 145-150
- [8] T. Kubina, J. Kliber, L. Kuncicka, M. Berkova, J. Horsinka, J. Boruta: *Metalurgija*, Vol. 52, 2013, No. 3, p. 325-328
- [9] I. Farup, J.M. Drezet, A. Mo, T. Iveland: *Journal of Thermal Stresses*, Vol. 23, 2000, p. 47-58, DOI: 10.1080/014957300280551
- [10] S. Spigarelli, E. Evangelista, H. J. McQueen: *Scripta Materialia*, Vol. 49, 2003, p. 179-183, DOI: 10.1016/S1359-6462(03)00206-9
- [11] J. Bidulská et al.: *Kovove Materialy*, Vol. 46, 2008, No. 3, p. 151-155
- [12] J. Bidulská, T. Kvačkaj, R. Bidulský, M. Cabibbo, E. Evangelista: *Metalurgija*, Vol. 46, 2007, No. 3, p. 157-159
- [13] A. Fedoriková, T. Kvačkaj, R. Kočiško, R. Bidulský, P. Petroušek: *Acta Physica Polonica A*, Vol. 131, 2017, No. 5, p. 1340-1343, DOI: 10.12693/APhysPolA.131.1340
- [14] El Mehtedi, M., Spigarelli, S.: *Acta Physica Polonica A*, Vol. 128, No. 4, 2015, p. 722-725, DOI: 10.12693/APhysPolA.128.722
- [15] F. Garofalo: *Fundamentals OF Creep and Creep-Rupture in Metals*. The Macmillan Company, New York, 1965
- [16] T. Kvačkaj, A. Kováčová, R. Kočiško: *Acta Physica Polonica A*, Vol. 128, 2015, No. 4, p. 689-692, DOI: 10.12693/APhysPolA.128.689
- [17] R. Pernis: *The Theory and Technology of Cups Production*, TnUAD Trenčín, Trenčín, 2009, (in Slovak)
- [18] J. Castellanos, I. Rieiro, M. Carsi, J. Muoz, O.A. Ruano: *Journal of Achievements in Materials and Manufacturing Engineering*, Vol. 19, 2006, No. 2, p. 56-63
- [19] F. Drastík, J. Elfmark: *Plastometers and forming of metals*. SNTL, Praha, 1977, (in Czech)
- [20] T. Kubina, A. Bořuta, M. Longauerová, J. Bořuta: *Materials Science Forum*, Vol. 782, 2014, p. 51-56, DOI: 10.4028/www.scientific.net/MSF.782.51
- [21] T. Kvačkaj, R. Kočiško, A. Kováčová: *Chemické listy*, Vol. 106, 2012, No. s3, p. s464-s467
- [22] K. Chadha, D. Shahriari, M. Jahazi: *La Metallurgia Italiana*, 2016, No. 4, p. 5-12
- [23] A. Hensel, T. Spittel: *The Force and Energy Consumption in Metal Forming Processes*, Deutscher Verlag für Grundstoffindustrie, Leipzig, 1978, (in German)
- [24] W.F. Hosford, R.M. Caddell: *Metal forming – Mechanics and Metallurgy*. Englewood Cliffs, Prentice-Hall, 1983
- [25] T.Z. Blazynski: *Plasticity and modern Metal-Forming Technology*. Elsevier Science Publishers, Barking-Essex, 1989
- [26] S. Rusz, I. Schindler, T. Kubina, J. Bořuta: *Acta Metallurgica Slovaca*, Vol. 12, 2006, No. 4, p. 477-483

### **Acknowledgements**

*Special thanks to my former colleague, Dr. Erika Hujová, who gladly translated this article from Slovak to English.*