

A LUMP-INTEGRAL MODEL FOR FREEZING AND MELTING OF A BATH MATERIAL ONTO A PLATE SHAPED SOLID ADDITIVE IN AN AGITATED BATH

Umesh Chandra Singh^{1)*}, Anant Prasad²⁾, Arbind Kumar³⁾

¹⁾ Engineering Division, Tata Steel Limited, Jamshedpur, India

²⁾ Retd, Department of Mechanical Engineering, National Institute of Technology, Jamshedpur, India

³⁾ Department of Mechanical Engineering, Birla institute of Technology Mesra, Ranchi, India

Received: 23.01.2012

Accepted: 26.11.2012

Corresponding author: e-mail: umeshchandra.singh@tatasteel.com, Tel: 091 0657 6648083, Engineering Division, Tata Steel Limited, Business Centre, Jamshedpur-831001, India.

Abstract

A lump integral model is developed for freezing and melting of the bath material onto the surface of a plate shaped additive immersed in an agitated melt bath. It exhibits the dependence of this occurrence on independent parameters-the initial temperature, θ_{ai} of the additive, the bath temperature, θ_b , the Biot number, B_i the property ratio, B and the Stefan number, S_i and yields closed-form solutions for time variant frozen layer thickness, ξ around the additive and heat penetration depth, η in the additive. In the solutions, B , B_i , θ_b and θ_{ai} appear as a conduction factor, C_{of} that ranges from 0 to ∞ . The frozen layer thickness per unit S_i with respect to C_{of} takes time $\tau_{cmax}=1/3$ for its maximum growth whereas this maximum thickness ξ_{cmax}^* becomes $(1-\theta_{ai})/3$. The total time of the growth of the maximum frozen layer thickness with its subsequent melting, τ_{ct} is $4/3$ when the heat penetration depth reaches the central axis of the plate additive, $\eta=1$. When $C_{of} \rightarrow 0$ signifying highly agitated bath ($h \rightarrow \infty$) or additive preheated to the freezing temperature of the bath material, no freezing of the bath material occurs. For the bath at the freezing temperature of the bath material, the frozen thickness is also obtained. The model is validated by reducing the present problem to heating of the plate additive subjected to a constant temperature maintained at the freezing temperature of the bath material.

Keywords: Mathematical modeling; melt-additive system; freezing, melting.

1 Introduction

Melting of a solid additive in a melt bath is employed in manufacture of steel, alloy, cast iron and similar other materials. It undergoes different steps. The first step is freezing and melting of the bath material around the surface of the additive along with rise in the temperature of the additive. In the second step, the additive is heated to its melting temperature after its emergence at an elevated temperature whereas it melts in the third step. These steps depend on the temperatures of the bath and the additive, bath agitation and thermo-physical properties of the additive- bath system and take certain time for their completion. Such a time regulates the productivity of manufacture of these materials. Since their increased productivity without compromise of quality for global competitiveness is of great importance, the reduction in the time of the production is essential. It can be achieved, once the time taken in the first step, which is not needed in the melting process of the additive but occurs due to requirement of heat to be

conducted in the additive more than the convective heat available from the bath during its initial period resulting in supply of latent heat of fusion by freezing of the bath material onto the additive and at its later period less than this available convective heat causing the excess of the convective heat to melt the frozen layer, is minimized. This is possible with the growth of a smaller thickness of the frozen layer. It is attained when the convective heat from the bath is increased by increasing the bath agitation. It essentially reduces the frozen layer thickness, the time of completion of the first step and does not allow the heat to penetrate the entire volume of the additive. Consequently, the total time of melting is decreased and the productivity of manufacturing is increased.

Investigation of such a situation that leads to thermal resistance of the frozen layer negligible with respect to that of the bath seldom appears in the literature. However, the occurrence of the first step for plate [1], cylindrical [2-4] and spherical [5, 6] shaped solid additives is analyzed when the frozen layer formed on these additives has their thermal resistances comparable with those of their bath. In this situation, it is observed that the increased heat transfer coefficient of the bath reduces the frozen layer thickness and time taken in the first step for the plate [1], spherical [6] and cylindrical [3] additives. This prediction is implicit in [2, 5] whereas in [4] only instant equilibrium temperature at the interface between the additive and the bath immediately after the immersion of the additive in the bath is found. Closed-form solutions for the growth of the maximum frozen layer thickness, its time of development and the total time of freezing and melting of the bath material onto the cylindrical additive in an agitated bath [7] is also reported recently.

In view of these facts, this work aims at development of a lump-integral model in dimensionless form for freezing and subsequent melting of the bath material onto the surface of the plate additive immersed in the agitated bath. The frozen layer formed is assumed to have much smaller thermal resistance than that of the additive. The model exhibits the dependence of this phenomenon on the independent parameters- initial temperature of the additive, θ_{ai} , the Stefan number, S_t indicative of phase change of the bath material, the Biot number, B_i representing the bath agitation, the property ratio, B and the bath temperature, θ_b . It provides closed-form solutions for frozen layer with subsequent melting, its completion time and heat penetration depth within the additive. They are functions of these independent parameters with B , B_i , θ_b and θ_{ai} occurring as a conduction factor, C_{of} . Further transformation makes these solutions dependent only on time. For this situation, time for maximum growth of frozen layer and total time for freezing of this layer with its subsequent melting are derived. To validate the model it is converted to a solution of the past investigation. A close agreement is exhibited.

2 Mathematical Model

To estimate the time for the first step as described in the introduction, a suitable mathematical model is designed. Here, the additive is in the form of a plate of thickness $2b$ and at a uniform temperature T_{ai} lower than its melting temperature, T_{af} before its immersion in the agitated melt bath contained in a ladle in which the bath is in a highly agitated state owing to falling stream of the melt during tapping or stirring in the ladle. This bath is assumed to remain at a constant temperature, T_b during the first step. Moreover, the plate additive has the melting temperature, T_{af} higher than the freezing temperature, T_{mf} of the bath material. For such a solid additive-melt bath system exhibited in **Fig.1**, $T_{ai} < T_{mf} < T_{af} < T_b$ and upon immersion of the plate additive, the melt of the bath immediately freezes onto the surface of the plate, the contact interface between

in a uniform temperature in the entire frozen layer thickness, d_m which is the freezing temperature, T_{mf} of the bath because the freezing front in contact with the bath always remains at T_{mf} . Moreover, the contact interface temperature, T_c between the plate and the frozen layer due to above fact is also at T_{mf} . These permit the frozen layer to act as a lump system [9-11] which due to remaining at T_{mf} does not absorb or release sensible heat. The conservation of energy to this lump leads to a balance between the sum of the latent heat of fusion evolved by freezing and convective heat supplied by the bath and the heat conducted to the additive. It takes the form

$$(d\xi/d\tau)/S_t + B_{im}(\theta_b - 1) = -Q_{mn}, \quad \xi_m = \xi, \tau > 0 \quad \text{with } BB_{im} = B_i \quad (5.)$$

It is subjected to an initial condition

$$\xi = 0, \tau = 0 \quad (6.)$$

The conjugated conditions at the interface between the additive plate and the frozen layer can be written as

$$(\partial\theta_a/\partial\xi_a)/B = -Q_{mn}, \quad \xi_a = \xi_m = 0, \tau > 0 \quad (7.)$$

$$\theta_a = \theta_m = \theta_e = 1, \quad \xi_a = \xi_m = 0, \tau > 0 \quad (8.)$$

Note that equations (1) to (8) form the mathematical model of the present problem. This type of model is identified as lump-integral since the frozen layer is assumed to be a lump and the additive is considered an integral system in the direction of heating. Governing equations (1) and (5) assume the uniform but different thermo-physical properties for the frozen layer and the plate additive whereas equations (7) and (8) are written when the surface of the plate is in perfect contact with the surface of the frozen layer with no interface resistance between them. These assumptions in the recent past investigation associated with freezing of the bath material onto the spherical [5, 6] and plate shaped additive [1, 12] having comparative thermal resistances of the bath with respect to the frozen layer provided acceptable solutions. These equations indicate the model dependence upon the independent parameters- initial temperature, θ_{ai} of the additive, the bulk temperature, θ_b of the bath, the modified Biot number, B_{im} and the Stefan number, S_t . It is noted that the B_i represents the condition of the bath whereas S_t is the indicative of the phase-change parameter of the bath material.

3 Solution

The present model exhibits that it is a moving phase-change problem owing to the presence of equation (5) and is coupled as a result of conjugating conditions, equations (7) and (8). These do not permit the model to give an analytical solution using exact analyses reported in the literature. In such a situation approximate analytical methods become of great significance and practical. One of these methods refer to integral method which yielded simple solutions in closed-form for melting or freezing and cooling or heating problems in previous investigations [13, 14] is applied. Also, in recently investigated other heating and phase-change problems [15-17], this method reduces them to initial value problems, the numerical solutions of which can be readily obtained using standard Runge-Kutta method. In view of this, the governing equation (1) for heating of plate additive has been already expressed in the integral format. For solution, it requires a prior knowledge of temperature field within the heated region of the additive which is assumed to be parabolic.

$$\theta_a = \theta_{ai} + (1 - \theta_{ai})(1 + \xi_a/\eta)^2 \quad (9.)$$

Note that such a profile is realistic since in the melting [15] and freezing [16] problems it yielded acceptable and accurate results. It satisfies the boundary conditions, equations (3) and (4). Employing equation (9), the integral equation (1) takes the form

$$\frac{d}{d\tau} \left[\theta_{ai}\eta + \frac{(1 - \theta_{ai})\eta}{3} \right] - \theta_{ai} \frac{d\eta}{d\tau} = \frac{1}{B} \left[\frac{2(1 - \theta_{ai})}{B\eta} - \left(\frac{\partial \theta_a}{\partial \xi_a} \right)_{\xi_a = -\eta} \right]$$

It reduces to

$$d[(1 - \theta_{ai})\eta/3]/d\tau = 2(1 - \theta_{ai})/(B\eta) \quad (10.)$$

once equation (4) is applied. Similarly, use of equation (9) in the conjugating condition, equation (7) gives.

$$2(1 - \theta_{ai})/(B\eta) = -Q_{mn} \quad (11.)$$

Substituting this equation in the energy balance equation (5) leads to

$$(d\xi/d\tau)/S_t + B_{im}(\theta_b - 1) = 2(1 - \theta_{ai})/(B\eta) \quad (12.)$$

It is noted that equation (12) for the frozen layer thickness, ξ gets coupled with equation (10) due to presence of η in it. These two equations (10) and (12) form simultaneous differential equation of first order in time, τ due to presence of a constant temperature difference $(1 - \theta_{ai})$ giving a closed-form solution. Their closed-form solutions are often difficult to obtain. However, their examination indicates that equation (10) can be reduced to

$$d\eta/d\tau = 6/(B\eta) \quad (13.)$$

Satisfying the initial condition, equation (2), it gives

$$\eta = \sqrt{12\tau/B} \quad (14.)$$

To solve equation (12), it is combined with equation (10) leading to

$$d[(1 - \theta_{ai})\eta/3 - \xi/S_t]/d\tau = B_{im}(\theta_b - 1) \quad (15.)$$

It readily gives a closed-form solution

$$(1 - \theta_{ai})\eta/3 - \xi/S_t = B_{im}(\theta_b - 1)\tau \quad (16.)$$

It fulfills the initial conditions, equations (2) and (6). This solution is not an explicit function of τ , rather functions of τ and η . To overcome this equation (14) is applied to make equation (16) a function of time.

$$\xi^* = \xi/S_t = -B_{im}(\theta_b - 1)\tau + (1 - \theta_{ai})\sqrt{12\tau/B}/3 \quad (17.)$$

Equations (17) and (14) give, respectively, closed-form relations for the growth of the frozen layer, ξ and the heat penetration depth η in the additive in terms of τ measured just after the immersion of the additive in the bath.

3.1 Maximum frozen layer thickness and its growth time

To find the development of the maximum frozen layer thickness, $d\xi^*/d\tau$ found from the equation (17) is made zero.

$$d\xi^*/d\tau = -B_{im}(\theta_b - 1) + (1 - \theta_{ai})\sqrt{12/B} \tau^{-1/2}/6 = 0 \quad (18.)$$

It gives

$$\tau/B = [(1 - \theta_{ai})/\{B_i(\theta_b - 1)\}]^2/3 = C_{of}^2/3 \quad (19.)$$

where,

$$B_i = B_{im} B = hb/K_a \quad \text{and} \quad C_{of} = (1 - \theta_{ai})/\{B_i(\theta_b - 1)\} = \{K_a(T_{mf} - T_{ai})/b\}/\{h(T_b - T_{mf})\} \quad (20.)$$

Here, the Biot number, B_i represents the ratio of the conductive resistance of the additive and the convective resistance of the bath whereas C_{of} , designated as conduction factor, is the ratio of the heat conducted to the additive, $K_a(T_{mf} - T_{ai})/b$ owing to the difference of the freezing temperature of the bath material and the initial temperature of the additive and the convective heat available from the bath, $h(T_b - T_{mf})$. Its values range from 0 to ∞ ($0 \leq C_{of} \leq \infty$). Zero indicates preheated additive at the freezing temperature of the bath material permitting no conductive heat transfer to the additive whereas ∞ signifies the bath at the freezing temperature of the bath material resulting non-availability of the convective heat from the bath. Using equation (19), equation (17) provides maximum frozen layer thickness

$$\xi_{max}^* = (1 - \theta_{ai})^2/[3B_i(\theta_b - 1)] = [(1 - \theta_{ai})C_{of}]^2/3 \quad (21.)$$

since $d^2\xi^*/d\tau^2 < 0$, the condition for the maximum value of ξ^* , is satisfied at such a time, which is hereafter called, τ_{max} .

3.2 Total time of freezing and subsequent melting of the bath material, τ_t

As in this time, τ_t the frozen layer, ξ^* developed melts completely giving $\xi^* = 0$. Its substitution in equation (17) provides an expression for τ_t

$$\sqrt{\tau_t} = [\sqrt{12/B}(1 - \theta_{ai})]/[3B_{im}(\theta_b - 1)]$$

It is rearranged to provide

$$\tau_t/B = (4/3)[(1 - \theta_{ai})/\{BB_{im}(\theta_b - 1)\}]^2 = (4/3)[(1 - \theta_{ai})/\{B_i(\theta_b - 1)\}]^2 = (4/3)C_{of}^2 \quad (22.)$$

Using equations (22) and (19), the time of melting, τ_m of the frozen layer can be written as

$$\tau_m = \tau_t - \tau_{max} = C_{of}^2 \quad (23.)$$

Note that equation (19) indicates that the time for the growth of the maximum thickness of the frozen layer ξ_{\max}^* is always 25% of the total time taken in freezing and its subsequent melting whereas equation (23) states that the melting of the frozen layer takes 75% of the total time.

3.3 Condition for the additive to act as semi-infinite

In several practical situations during the period, τ_t of freezing and melting of the bath material, the additive plate does not get completely heated owing to which the heat does not penetrate the central axis of the plate. In this condition the plate acts as semi-infinite and the heat penetration depth, η within it always remains $\eta \leq 1$. Applying it to equation (14) leads to

$$\tau_t/B \leq 1/12 \quad (24.)$$

Substitution of the equation (24) in equation (22) provides

$$C_{of} \leq 1/4 \quad (25.)$$

It is the requisite condition for the additive to remain semi-infinite during freezing and melting of the bath material.

3.4 Reduction of solutions only as a function of time

Since equations (19) to (23) are functions of conduction factor, C_{of} , they including equations (14) and (17) are transformed with respect to this factor reducing them to only a function of time. Using this concept, the growth of frozen layer thickness, equation (17) and the heat penetration depth equation (14) become respectively

$$\xi_{cm}^{**} = \xi_{cm}^* / (1 - \theta_{ai}) = \sqrt{12\tau_{cm}} / 3 - \tau_{cm} \quad (26.)$$

$$\eta_{cm} = \sqrt{12\tau_{cm}}, \quad (27.)$$

where, $\xi_{cm}^* = \xi^* / C_{of}$, $\tau_{cm} = \tau / (BC_{of}^2)$, $\eta_{cm} = \eta / C_{of}$

whereas time of maximum frozen layer thickness equation (19) and total time of freezing and melting equation (22) can be expressed respectively as

$$\tau_{cm} = \tau / (BC_{of}^2) = 1/3 \quad (28.)$$

$$\tau_{ct} = \tau_t / (BC_{of}^2) = 4/3 \quad (29.)$$

The growth for the maximum frozen layer thickness, equation (21) is changed to

$$\xi_{c \max}^{**} = \xi_{c \max}^* / (1 - \theta_{ai}) = 1/3, \quad \text{where, } \xi_{c \max}^* = \xi_{\max}^* / C_{of} \quad (30.)$$

Although the freezing and melting of the bath material onto the surface of plate additive follows a complicated heat transfer process, this model enables to make the frozen layer growth, equation (26) and the heat penetration depth, η equation (27) only a function of time.

4 Model Validation

To validate the present model, this problem is transformed to heating of the additive by the bath when no freezing of the bath material occurs onto the surface of the plate additive. This can be achieved once the bath is assumed to be highly agitated with heat transfer co-efficient $h \rightarrow \infty$, Biot number, $B_i \rightarrow \infty$ or the latent heat of fusion of the bath material is very high $L_m \rightarrow \infty$, $S_t \rightarrow 0$ leading to growth of the frozen layer of insignificant thickness, $\xi \rightarrow 0$. The contact interface temperature between the insignificant frozen layer and the plate additive remains at the freezing temperature, T_{mf} of the bath material. These reduce the present problem of heating of the plate at a constant temperature, T_{mf} . These concepts vanish equation (17) without altering equation (14) providing

$$\eta = D\tau_h^{1/2}, \text{ where, } D = \sqrt{12} = 3.46 \quad (31.)$$

The exact solution [18] gives $D = 3.66$ whereas the variational method [19] provided $D = 3.36$. A close agreement is observed validating the present model.

5 Results and Discussions

The mathematical model of lump-integral form just developed for freezing and melting of the bath material around the surface of the plate shaped additive immersed in an agitated bath shows that this occurrence is regulated by the initial temperature, θ_{ai} of the additive, the bath temperature, θ_b , the bath agitation represented by the Biot number, B_i , phase-change parameter of the bath material denoted by the Stefan number, S_t and the property-ratio, B of the additive-bath system. However, in the closed-form solutions θ_{ai} , θ_b , B , and B_i appear as a conduction factor, C_{of} , equation (20). For different plate shaped additive-bath systems employed in steelmaking and cast iron preparation, values of these parameters appear in Table1. The conduction factor, C_{of} ranges from 0 to ∞ . $C_{of} = 0$ signifying no conductive heat transfer to the additive which, in turn, does not permit freezing of the bath material onto the additive leading to elimination of the first step of the freezing and melting. In practice it can be achieved by preheating the additive to the freezing temperature of the bath material ($\theta_{ai} = 1$) before its immersion in the bath or increasing the bath agitation for a prescribed temperature, θ_{ai} of the additive. $C_{of} = \infty$ indicates that the convective heat is not available from the bath. It happens when the bath temperature is maintained at the freezing temperature of the bath material ($\theta_b = 1$). Under this situation, the conductive heat required by the additive after its immersion in the bath is met by only evolution of latent heat of fusion due to freezing of the bath material. Consequently, the frozen layer continues to grow and melting of this layer never takes place. Due to these facts a smaller value of C_{of} close to zero is preferred for growth of a small frozen layer thickness so that much less time is required for freezing with its melting. The Stefan number, S_t is the ratio of sensible heat and latent heat of fusion of the bath material. Its high value represents bath material of small latent heat of fusion leading to growth of large thickness of the frozen layer. In the closed-form solution for the frozen layer thickness it is taken per unit S_t , equation (17) termed as ξ^* for making it applicable to any bath material. Further, ξ^* with respect to the conduction factor C_{of} called ξ_{cm}^* becomes only a function of the initial temperature, θ_{ai} of the additive, equation (26). The heat penetration depth, η_{cm} equation (27) is implicitly dependent on these factors. In these formats, the growth of ξ_{cm}^* to its maximum thickness takes time $\tau_{cm} = 1/3$ whereas total time τ_{ct} for freezing to this maximum thickness and its melting is $4/3$. The maximum thickness of the frozen layer assumes $\xi_{c\max}^* = (1 - \theta_{ai})/3$. These

forms have advantage of applying to any plate shaped additive and the bath materials. For the additive-bath systems of Table 1 with the plate additive of given semi-thickness and initial temperature, total time of freezing and melting along with the maximum thickness of the frozen layer developed can be readily obtained employing equations (22) and (21), respectively.

Table 1 Thermo-physical properties of the Bath materials [1,5,20].

Bath material	K_m	ρ_m	C_{pm}	$L_m \times 10^{-3}$	T_{mf}	T_b
	$Wm^{-1}K^{-1}$	Kgm^{-3}	$JKg^{-1}K^{-1}$	JKg^{-1}	$^{\circ}C$	$^{\circ}C$
Cast-Iron ~4%C	51.9	7304	417	275.7	1160	1550
Hot-Metal	35	6850	670	275.7	1150	1500
Slag	1.06	2890	920	544	1500	1600

Table 2 Thermo-physical properties of the Solid additive [1,5,20].

Solid additive	K_a	ρ_a	C_{pa}	T_{ai}	$b \times 10^3$
	$Wm^{-1}K^{-1}$	Kgm^{-3}	$JKg^{-1}K^{-1}$	$^{\circ}C$	m
Ni	90	8906	449.5	20	5
				40	125
Ferro-Magnese	7.5	7200	700	20	5
				40	125
DRI	2.13	2600	820	20	5
				40	125

Table 3 Non-dimensional parameters of Bath-solid additive system [1,5,20].

Bath-solid additive system	$\theta_{ai} \times 10^3$	θ_b	B	$*B_i$	C_{of}	S_t
Cast iron ~4%C and Ni	17.2	1.33	0.44	0.33	8.77	1.75
	34.5			8.33	0.34	
Hot metal and Ferro-Magnese	17.4	1.3	4.25	4	0.81	2.79
	34.8			100	0.03	
Slag and DRI	13.3	1.06	0.62	14.08	1.05	2.54
	26.6			352.1	0.04	

*Based on heat transfer co-efficient, $h = 6000 Wm^{-2}k^{-1}$

5.1 Effect of conduction factor, C_{of}

Displayed in Fig.2 are time variant growth of frozen layer thickness per unit S_t , ξ^* and the heat penetration depth, η in the additive for a prescribed initial temperature, θ_{ai} of the additive. The conduction factor, C_{of} is taken as a parameter. It indicates that for each C_{of} the freezing and melting assumes a parabolic behavior with the height of the apex of the parabola signifying the maximum thickness of the frozen layer. Decreasing C_{of} decreases the height of the apex of the parabola and in turn, the maximum thickness of the frozen layer. Moreover, the apex shifts towards zero time and both the total time of freezing and melting, equation (22) and the time taken to attain the maximum thickness, equation (19) decrease with square of C_{of} . The heat penetration depth at the time of completion of the freezing and melting also diminishes. These findings appear to be realistic since for a prescribed initial temperature, θ_{ai} of the additive allowing a certain amount of heat to be conducted to the additive, the reduction in C_{of} increases

the availability of convective heat from the bath for a given bath temperature. This increases the Biot number and makes the bath more agitated. Also, as the heat conducted to the additive is sum of the convective heat available from the bath and the latent heat of fusion liberated owing to the freezing of the bath material onto the additive during the development of the frozen layer, increased convective heat from the bath by the reduced C_{of} requires less amount of heat liberated as the latent heat of fusion to meet the above desired conductive heat. It leads to a growth of smaller thickness of the frozen layer, **Fig.2**.

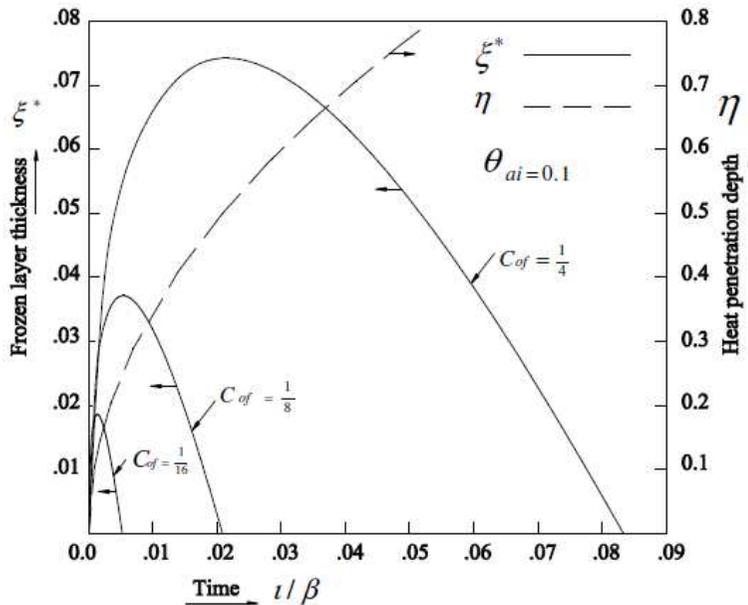


Fig. 2 Effect of C_{of} , on time dependent frozen layer thickness, heat penetration depth and total time of freezing and melting for additive at Θ_{ai}

5.2 Influence of initial temperature of the additive, θ_{ai}

Illustrated in **Fig.3** are the frozen layer thickness, ξ^* and the heat penetration depth, η as functions of time for different values of initial temperature, θ_{ai} of the additive. They are for a particular conduction factor, C_{of} . Although the freezing and melting follows the parabolic behavior for each θ_{ai} , with increasing initial temperature, θ_{ai} , the maximum frozen layer thickness, equation (21) decreases, but the time taken to attain this thickness, equation (19), total time of freezing and melting, equation (22) and the heat penetration depth remain unaltered. When the initial temperature, θ_{ai} of the additive is at the freezing temperature of the bath material, no freezing occurs. The following facts support these predictions. For a specified value of conduction factor, C_{of} decreasing θ_{ai} increases the heat conduction to the additive and in order to maintain this value of C_{of} , the convective heat from the bath is to be increased to the amount by which the heat conduction is increased. As a result, the requirement of latent heat of fusion is increased leading to formation of larger thickness of the frozen layer. Moreover, as the value of C_{of} , a ratio of conductive heat transfer to the additive and the convective heat supplied by the bath does not change despite the change in the values of these two heat transfer due to change in θ_{ai} , the requirement of time for development of the frozen layer with its subsequent melting

remains the same. The growth of the frozen layer and its melting, however, gets speed up owing to an increased temperature difference between the bath and the additive once the initial temperature, θ_{ai} of the additive is decreased.

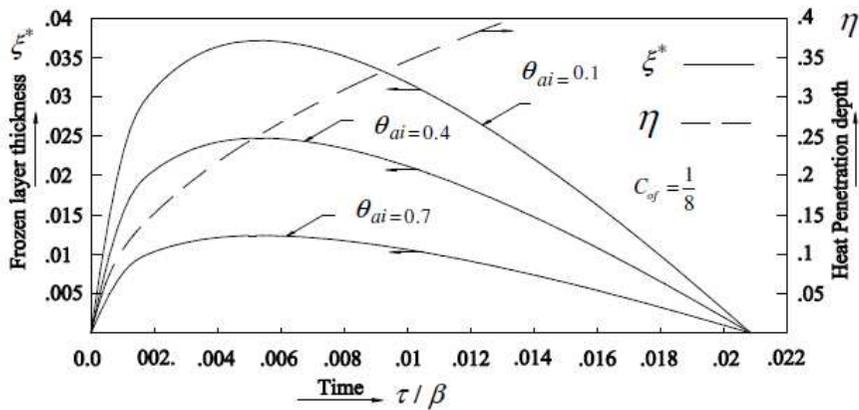


Fig. 3 Influence of initial temperature, θ_{ai} of additive on time of growth of frozen layer, total time of freezing and melting and heat penetration depth for a given conduction factor, C_{of} of bath-additive system

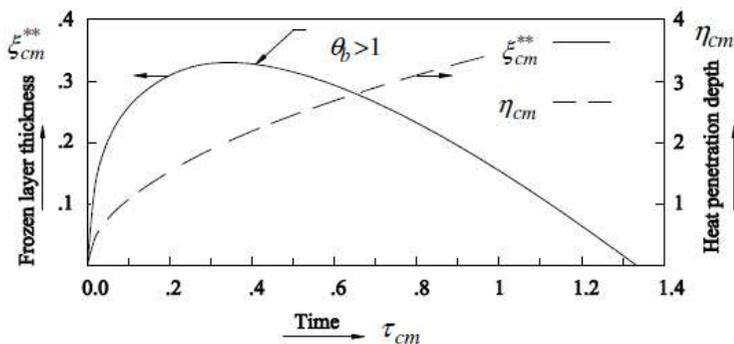


Fig. 4 Time for development of frozen layer, heat penetration depth including total time of freezing and melting. They are for any plate additive having any initial temperature

5.3 Growth of frozen layer applicable to any plate additive and bath materials

With respect to the conduction factor, C_{of} , ξ^* per unit difference of temperature between the freezing temperature of the bath material and the initial temperature, θ_{ai} of the additive represented by ξ_{cm}^{**} , equation (26) and the heat penetration depth denoted by η_{cm} equation (27) become only the function of time. Their plots with time, **Fig.4** assume the behavior similar to those appear in Figs.2 and 3. They act as universal graphs and applied to any plate shaped additive-bath materials provided the freezing and subsequent melting of the bath material onto the additive takes place. In this situation, the maximum frozen layer thickness, $\xi_{cm}^{**} = 1/3$, equation (30) with its time of formation, $\tau_{cm} = 1/3$, equation (28) whereas the total time of freezing of this thickness with its subsequent melting is $\tau_{ct} = 4/3$. For any of the additive-bath system contained in Table1, these graphs or equations (30), (28), (29) readily give the

dimensional values of frozen layer thickness, time for its maximum growth and total time of freezing and melting once the data of the Table 1 is employed. They are beneficial for industrial applications.

6 Conclusions

A non-dimensional lump-integral model evolved for the first step of the freezing and melting of the bath material onto the surface of the plate shaped additive in an agitated bath gives closed-form solutions for the growth of the frozen layer with its subsequent melting and the heat penetration depth in the plate additive with the time measured immediately after their occurrence. They are regulated by conduction factor, C_{of} and θ_{ai} . Decreasing C_{of} reduces the growth of maximum thickness of the frozen layer, its time of growth and the total time of freezing with its melting. Increasing the initial temperature of the additive, θ_{ai} the growth of the maximum frozen layer thickness reduces. However, its time of growth and total time of freezing with its melting almost remains the same. This step almost disappears and the frozen layer thickness becomes almost zero once the bath is assumed to be highly agitated $C_{of} \rightarrow 0$. With respect to C_{of} , these solutions lead to frozen layer and the heat penetration depth only a function of time which are applied to any additive-bath system provided the additive is plate shaped. In this situation, the time taken for the maximum thickness of the frozen layer, ξ_{cmax}^{**} is $\tau_{cmax}=1/3$ whereas the total time taken for the first step is $\tau_{ct}=4/3$.

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Appendix:

- b semithickness of the plate, m
- B property ratio, $(K_m C_m / K_a C_a)$
- B_i Biot number, (hb / K_a)
- B_{im} Modified Biot no., $(hR_0 / K_a) * (K_a C_a / K_m C_m)$
- C heat capacity (ρC_p) , $Jm^{-3}K^{-1}$
- C_p Specific heat, $JKg^{-1}K^{-1}$
- C_{of} conduction factor, $(1 - \theta_{ai}) / B_i (\theta_b - 1)$
- d heat penetration depth or frozen layer thickness at any time, m
- h heat transfer coefficient, $Wm^{-2}K^{-1}$
- K thermal conductivity, $Wm^{-1}K^{-1}$
- L latent heat of fusion, JKg^{-1}
- Q_m heat transfer from the frozen layer to the additive, Wm^{-2}
- Q_{mn} non-dimensional heat transfer from the frozen layer to the additive, $(Q_m b / K_a T_{mf}) / B$
- S_t Stefan number, $(C_m T_{mf} / L_m \rho_m)$
- t time, s
- T temperature, K
- x distance along the heat penetration depth or frozen layer, m

Greek letters

- α thermal diffusivity, m^2s^{-1}
- ξ non-dimensional thickness of the frozen layer, $(C_m d_m / C_a b)$ or distance along frozen layer, $(C_m x_m / C_a b)$ or the additive (x_a / b)
- η non-dimensional heat penetration depth in the additive at any time, (d_a / b)
- ρ density, kgm^{-3}
- θ non-dimensional temperature, (T / T_{mf})
- τ non-dimensional time, $(K_m C_m / C_a^2 b^2) t$

Subscript

- a plate additive
- ai initial condition of additive
- b bulk condition
- e at the interface between the additive and the frozen layer at time.
- m frozen bath material