THE ALUMINIUM PRICE FORECASTING BY REPLACING THE INITIAL CONDITION VALUE BY THE DIFFERENT STOCK EXCHANGES

Marcela Lascsáková[1)], Peter Nagy[1)]*
[1)] *Technical University of Košice, Faculty of Mechanical Engineering, Košice, Slovakia*

Received: 11.04.2013
Accepted: 01.08.2013

**Corresponding author: e-mail: marcela.lascsakova@tuke.sk, Tel.: +421 55 602 2219, Department of Applied Mathematics and Informatics, Faculty of Mechanical Engineering, Technical University of Košice, Letná 9, 042 00 Košice, Slovakia*

Abstract
In mathematical models, for forecasting prices on commodity exchanges different mathematical methods are used. In the paper the numerical model based on the exponential approximation of commodity stock exchanges was derived. The price prognoses of aluminium on the London Metal Exchange were determined as numerical solution of the Cauchy initial problem for the 1st order ordinary differential equation. To make the numerical model more accurate the idea of the modification of the initial condition value by the aluminium price (stock exchange) was realised. The derived numerical model was verified by determining the influence of the length of the initial condition drift on the accuracy of the obtained prognoses. The types of the initial condition drift during different movements of aluminium prices were studied. The most accurate prognoses were the most often obtained by using the longest initial condition drift. In this type of drift the initial condition value was replaced by aluminium stock exchange in the month in which the absolute percentage error of the prognosis had at least selected value. The advantage of this drift was manifested especially in the stable price course and within larger changes in prices. If there was price fluctuating within the observing period, in the next forecasting the most accurate was the drift the initial condition value of which had been replaced by the price that was the nearest to the stock exchange price evolution.

Keywords: aluminium, exponential approximation, numerical modelling, price forecasting, commodity exchange

1 Introduction
Observing trends and forecasting movements of metal prices is still a current problem. There are a lot of approaches to forecasting price movements [1, 2]. Some of them are based on mathematical models [2-19]. Forecasting prices on commodity exchanges often uses the statistical methods that need to process a large number of historical market data [5-12]. The quantity of needed market data can sometimes be a disadvantage. In such cases, other mathematical methods are required.
In our prognostic model numerical methods were used. Their advantage is that, in comparison with statistical models, many fewer market data are needed. Our numerical model for forecasting prices is based on the numerical solution of the Cauchy initial problem for the 1st order ordinary differential equations [13-19].
The aluminium prices presented on the London Metal Exchange (LME) were worked on. We dealt with the monthly averages of the daily closing aluminium prices "Cash Seller&Settlement

DOI 10.12776/ams.v20i1.183 p-ISSN 1335-1532
e-ISSN 1338-1156

price" in the period from December 2002 to June 2006. The market data were obtained from the official web page of the London Metal Exchange [20]. The course of the aluminium prices on LME (in US $ per tonne) within the observing period is presented in **Fig. 1**. As we can see in **Fig. 1** the course of the aluminium prices within the considered period changes dramatically.

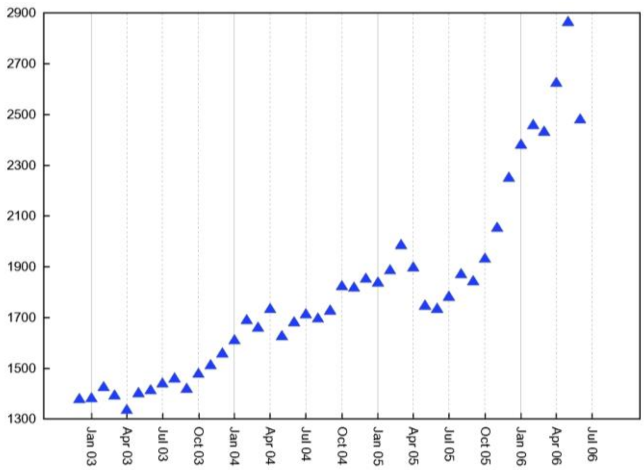

Fig. 1 Course of aluminium prices on LME in the years 2003 - 2006

We started from the original model calculating the prognoses within six months following the approximation term after modification of the initial condition value by the obtained monthly price prognoses [17-19]. The original model forecasts the aluminium price reliably within the stable price course, when the price does not changed rapidly. Within the rapid increase or decrease of stock exchanges, but also in the case of changes in the price course the forecasting fails. Since the variability with rapid and sudden changes is typical of the commodity price course, we judged the possibility of making the forecasting more accurate by using the modification of the initial condition value by aluminium price. We analysed the use of the initial condition drift concerning differently chosen stock exchanges for successful forecasting.

2 Mathematical model

The Cauchy initial problem in the form

$$y' = a_1 y, \quad y(x_0) = y_0 \tag{1}$$

is considered. The particular solution of the problem (1) is in the form $y = k\, e^{a_1 x}$, where $k = y_0\, e^{-a_1 x_0}$. The considered exponential trend was chosen according to the test criterion of the time series' trend suitability. The values $\ln(Y_{i+1}) - \ln(Y_i)$, for $i = 0,1,\ldots,42$ have approximately constant course. (Y_i is the aluminium price (stock exchange) on LME in the month x_i.)

The price prognoses are created by the following steps:

1st step: Approximation of the values – the values of the approximation term are approximated by the least squares method. The exponential function in the form $\tilde{y} = a_0 e^{a_1 x}$ is used. When observing the influence of the approximation term length on the prognoses accuracy, we found out that the prognoses obtained by longer approximation terms are more accurate [14, 18]. The approximation of larger number of the values is not subject to abrupt changes and follows a longer price course evolution. Let us consider two different variants:

DOI 10.12776/ams.v20i1.183 p-ISSN 1335-1532
e-ISSN 1338-1156

Variant B: The values from the period January 2003 - June 2003 are approximated. The next approximation terms are created by sequential extension of this period by 3 months. Thus the duration of the approximation terms is extended (the n^{th} approximation term has $6+3(n-1)$ stock exchanges) (**Fig. 2**).

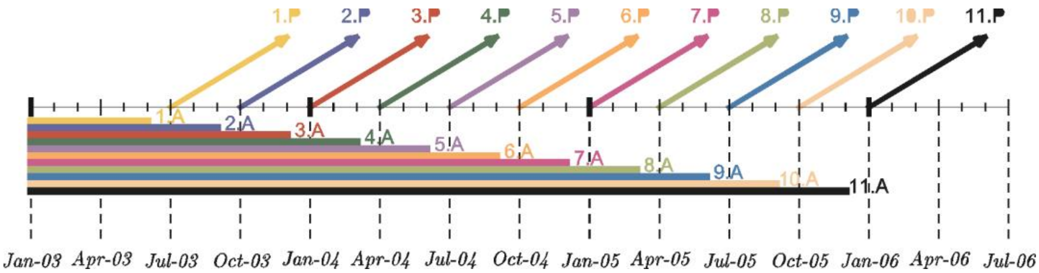

Fig. 2 Variant B (A – approximation term, P – forecasting term) (as can be seen in [17-19])

Variant E: We approximate values within 12 months and each term is shifted by 1 month (**Fig. 3**). (The first approximation term is January 2003 - December 2003.)

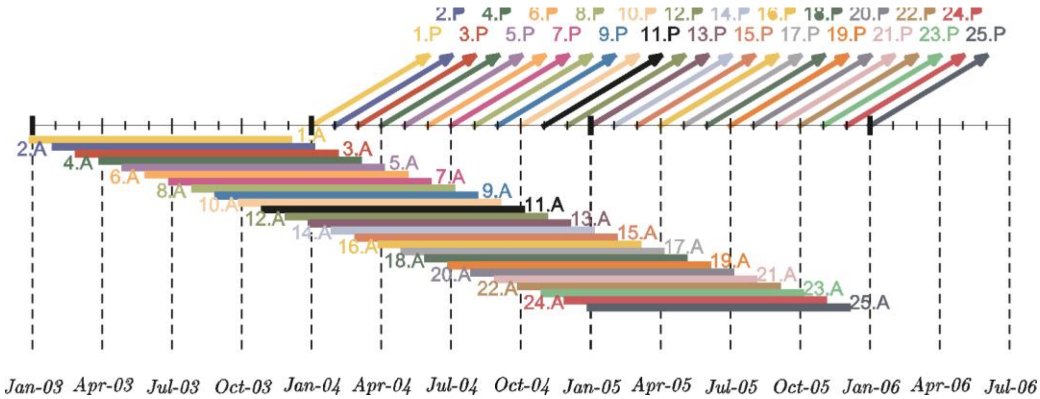

Fig. 3 Variant E (A – approximation term, P – forecasting term) (as can be seen [17-19])

2nd step: Formulating the Cauchy initial problem – according to the acquired approximation function $\tilde{y}$, the Cauchy initial problem (1) is written in the form

$$y' = a_1 y, \quad y(x_i) = Y_i, \tag{2}$$

where $x_i = i$ is the last month of the approximation term,
Y_i is the aluminium price on LME in the month x_i.

3rd step: Computing the prognoses – the formulated Cauchy initial problem (2) is solved by the numerical method based on the exponential approximation of the solution. A detailed solution method is seen in [21]. The method uses the following numerical formulae:

$$x_{i+1} = x_i + h,$$

$$y_{i+1} = y_i + bh + Qe^{vx_i}\left(e^{vh} - 1\right),$$

DOI 10.12776/ams.v20i1.183 p-ISSN 1335-1532
e-ISSN 1338-1156

for $i = 1, 2, 3, \ldots$, where $h = x_{i+1} - x_i$ is the constant size step.
The unknown coefficients are calculated by means of these formulae:

$$v = f''(x_i, y_i) / f'(x_i, y_i),$$

$$Q = \left(f'(x_i, y_i) - f''(x_i, y_i) \right) / \left((1 - v)\, v^2 e^{v x_i} \right),$$

$$b = f(x_i, y_i) - f'(x_i, y_i)/v.$$

If we consider the Cauchy initial problem (2), the function $f(x_i, y_i)$ has the form $f(x_i, y_i) = a_1 y_i$ and then $f'(x_i, y_i) = a_1 y'(x_i) = a_1^{\,2} y_i$, $f''(x_i, y_i) = a_1^{\,2} y'(x_i) = a_1^{\,3} y_i$.
We calculate the prognoses within six months that follow the end of the approximation term in this way:

- **The first month prognosis** is determined by solving the Cauchy initial problem in the form (2). The interval $\langle x_i, x_{i+1} \rangle$ of the length $h = 1$ month is divided into n parts, where n is the number of trading days on LME in the month x_{i+1}. We get the sequence of the division points $x_{i0} = x_i$, $x_{ij} = x_i + (h.j)/n$, for $j = 1, 2, \ldots, n$, where $x_{in} = x_{i+1}$. For each point of the subdivision of the interval, the Cauchy initial problem in the form (2) is solved by the chosen numerical method. In this way we obtain the prognoses of the aluminium prices on single trading days y_{ij}. By calculating the arithmetic mean of the daily prognoses we obtain the monthly prognosis of the aluminium price in the month x_{i+1}. So, $y_{i+1} = \sum_{j=1}^{n} y_{ij} / n$.
- **The prognoses for the following months** shall be calculated after modification of the initial condition value. The initial condition value in the month x_{i+s}, $s = 1, 2, 3, 4, 5$ is replaced either by the calculated monthly prognosis y_{i+s} or by some aluminium stock exchange (in case of higher absolute percentage error of given monthly prognosis y_{i+s}). The Cauchy initial problem $y' = a_1 y,\ y(x_{i+s}) = y_{i+s}$, respectively $y' = a_1 y,\ y(x_{i+s}) = Y_p$ (where Y_p is chosen aluminium stock exchange) is used for calculating daily prognoses and their arithmetic mean serves to define the monthly price prognosis y_{i+s+1} for the month x_{i+s+1}.

By comparing the calculated prognosis y_s in the month x_s with the real stock exchange Y_s, the absolute percentage error $|p_s| = \left(|y_s - Y_s| / Y_s \right).100\,\%$ is determined. The price prognosis y_s in the month x_s is acceptable in practice, if $|p_s| < 10\,\%$. Otherwise, it is called the critical forecasting value of. To compare the accuracy of forecasting of all forecasting terms, the mean absolute percentage error (MAPE) $\bar{p} = \sum_{s=1}^{t} |p_s| / t$ is determined, where, in our case, $t = 6$.
The modification of the initial condition value by the real aluminium stock exchange price is called the initial condition drift. Let us name the selected minimal absolute percentage error of the prognosis, causing the initial condition drift, the limiting value error. The month in which the absolute percentage error of the prognosis has at least the limiting value error will be considered as the limiting month. We have chosen the limiting value error of 7%. Let us consider three different types of the initial condition drift:

- **1-month drift,**
- **drift before the limiting month,**
- **drift to the limiting month.**

3 Results

3.1 The original model – the model without using initial condition drift

In the original model the prognoses in the months x_{i+s}, $s = 2, 3, 4, 5, 6$ (where x_i is the last month of the approximation term) are calculated after modification of the initial condition value

DOI 10.12776/ams.v20i1.183 p-ISSN 1335-1532
e-ISSN 1338-1156

by the obtained monthly price prognoses. 36 forecasting terms of the original model in both variants B and E within period from July 2003 to June 2006 are observed. From among all forecasting terms, 11 of them belong to variant B and 25 ones are part of variant E, whereby 9 forecasting terms are common for both variants. In the observing period we gained 216 original monthly price prognoses. Based on the prognosis accuracy analysis of the original model, we classified forecasting terms into the following classes:

I. trouble free forecasting terms (18 terms)

All absolute percentage errors of the monthly prognoses within 6-month forecasting term are $<10\ \%$;

a) The initial condition drift does not occur (14 terms) – either all absolute percentage errors of the monthly prognoses are $<7\ \%$ or the absolute percentage prognosis error in the last month of the forecasting term is from the interval $\langle 7,10)$.

b) The initial condition drift occurs (4 terms) – the absolute percentage monthly prognoses errors in some months within observed forecasting term are higher than or equal to $7\ \%$;

II. forecasting terms with a small error (10 terms)

The mean absolute percentage error of the forecasting term is less than $10\ \%$, but the absolute percentage errors of some monthly prognoses are at least $10\ \%$ (there are the critical forecasting values in forecasting term);

III. forecasting terms with a big error (8 terms)

The mean absolute percentage error of the forecasting term is $\geq 10\ \%$.

From among 36 observed forecasting terms, a half of them are trouble free. Approximately in 3/4 of these terms, the forecasting is so accurate that the initial condition drift does not occur. The initial condition values are replaced only by calculated monthly prognoses; thereby the original model has not changed. The second half of the original terms consists of the forecasting terms with different errors causing the initial condition drift. This explains why the forecasting results differ from the original model. The forecasting terms with small and big errors are almost equally met.

By having analysed the aluminium price evolution we found out that the trouble free forecasting terms are typical of a moderate price increase with its occasional oscillation. Within the forecasting terms with a small error, the price increase in the forecasting term is more rapid than the price increase in an approximation term, or there is a rapid decrease of the stock exchanges within the forecasting term, even though at its beginning there is aluminium price increase. In the forecasting terms with a big error we can observe a rapid aluminium price decrease, or increase, immediately after a period of price decline.

3.2 The influence of the initial condition drift length on the accuracy of the forecast prices

The forecasting terms in which the initial condition drift occurred (22 terms) were taken into consideration. We were interested in whether different initial condition drift length affected the forecasting accuracy. Let us consider three types of the initial condition drift with regard to their length:

1. **1-month drift,**
2. **drift before the limiting month,**
3. **drift to the limiting month.**

For each forecasting term we defined the type of drift with the lowest MAPE. We also observed the ability of initial condition drift to eliminate the critical forecasting values. The following

DOI 10.12776/ams.v20i1.183 p-ISSN 1335-1532
e-ISSN 1338-1156

table shows the number of the forecasting terms in which the forecasting by the determined types of the initial condition drift was the most accurate.

Table 1 The comparison of the success rate of the chosen types of the initial condition drift for improving the prognoses accuracy of the original model

Type of forecasting term	Type of drift		
	1-month drift	drift before the limiting month	drift to the limiting month
I.	1	2	2
II.	3	0	8
III.	0	0	8

With regard to the initial condition drift length, the most accurate forecasting results were obtained by using the drift to the limiting month, in other words, the longest initial condition drift. Using it, we move really close to the real forecast prices. This type of drift, in comparison with the other two drifts, had the lowest MAPE in 18 forecasting terms. In two of them we obtained the same results by using 1-month drift (the initial condition drifts were the same). The 1-month drift was far less successful. It was the most advantageous type for 4 forecasting terms, 2 of them are already mentioned in the previous drift type. The drift before the limiting month was the most accurate only in 2 forecasting terms.

Let us analyze the success rate of the determined types of the initial condition drift within different moves of the aluminium price course:

- **unstable moderate price increase**

The unstable moderate increase of the stock exchanges can be seen in the forecasting terms of the years 2003, 2004 and at the beginning of the year 2005. Within these terms, the original forecasting was mostly acceptable (the absolute percentage of prognoses errors were $< 10\ \%$), so the forecasting terms belong to the trouble free forecasting terms. In most of these terms, the absolute percentage errors were even $< 7\ \%$, so the initial condition drift did not occur. In the trouble free forecasting terms, bigger prognoses errors are given by the change of the increase rate of the forecast stock exchanges in comparison with the stock exchanges within the approximating term. The initial condition drift occurred only in three different forecasting terms. In each term, forecasting by using different initial condition drift length was the most successful. With regard to the moderate course of the stock exchanges, the initial condition drift to the values appropriate for the next stock exchanges course was the most advantageous.

- **steep price decrease**

There was steep price decrease from April 2005 to June 2005. Within this period, the price decrease was significant compared to the previous increase. The forecasting terms within these months can be divided into two groups:

a) periods of the price increase with the consecutive price decline;
b) periods of the price decline with the consecutive unstable increase of the stock exchanges.

In both groups of the forecasting terms, the stock exchanges in approximation terms are increasing, the approximation functions have also an increasing course, and the prognoses

DOI 10.12776/ams.v20i1.183 p-ISSN 1335-1532 e-ISSN 1338-1156

calculated by the original model are increasing too, but they are not sufficient to accommodate to a steep decline of the stock exchanges. The longer is the decline period, the higher is the absolute percentage of prognosis error. In the periods when the price decline includes a smaller part of the forecasting term, forecasting is trouble free even without the initial condition drift. The periods in which larger part includes the price decline belong to the forecasting terms with a small or big error. In all these terms, the drift to the limiting month was the most accurate. By using the longest drift, the initial condition value was the nearest to the decline prices. Thus, the following critical values were eliminated, and the forecasting became much more accurate. As the increase of the stock exchanges occurred in the next period, the forecasting was successful and no coercions were needed. The greatest forecasting improvement was obtained in the forecasting term with a big error *April 2005 – September 2005*. The mean absolute percentage error in this term was decreased by using the initial condition drift to the limiting month from 12, 55 % to 4, 96 % (variant B) and from 12, 63 % to 4, 94 % (variant E). The number of critical values in both variants was reduced to one critical value against five critical values within the original forecasting.

- **unstable moderate price increase after price decline followed by rapid increase**

The end of the year 2005 and the first half of the year 2006 appear as the most problematic. In this period there are 5 forecasting terms with a small error and 6 forecasting terms with a big error. The problems in forecasting are caused by the steep increase of the stock exchanges after their important decline. The approximation terms with the price decline belong to the mentioned forecasting terms. The higher is the number of the decline prices in approximation term, the slower is increase of the approximation function. Its course could be even decreasing, as shown in **Table 2**. As these approximation functions serve for the price prognoses within the rapid increase periods, the original forecasting being highly inaccurate. More important changes in the course of approximation functions can be seen in the variant E in which, in comparison with the variant B, fewer values are approximated. Thus the course of the approximation functions of the variant E is more affected by the price decline.

Table 2 Summary of approximation functions for increasing forecasting terms after aluminium price decline in 2005

Forecasting term	Variant B	Variant E
June 2005 - November 2005		$\tilde{y} = 1414{,}3e^{0{,}0103x}$
July 2005 - December 2005	$\tilde{y} = 1340\,e^{0{,}0122x}$	$\tilde{y} = 1574{,}2e^{0{,}0056x}$
August 2005 - January 2006		$\tilde{y} = 1696{,}1e^{0{,}0026x}$
September 2005 - February 2006		$\tilde{y} = 1792{,}2e^{0{,}0007x}$
October 2005 - March 2006	$\tilde{y} = 1357{,}3e^{0{,}0110x}$	$\tilde{y} = 1918{,}2e^{-0{,}0016x}$
November 2005 - April 2006		$\tilde{y} = 1848{,}3e^{-0{,}00008x}$
December 2005 - May 2006		$\tilde{y} = 1692{,}6e^{0{,}0033x}$
January 2006 - June 2006	$\tilde{y} = 1348{,}5\,e^{0{,}0115x}$	$\tilde{y} = 1390{,}4e^{0{,}0101x}$

Slower increasing prognoses obtain lower values than quicker increasing stock exchanges; this is why the forecasting accuracy is decreasing with time (critical values applied to the months at the end of the forecasting terms). The forecasting was the most accurate when using the longest drift. By applying it, we put prognoses the nearest to the steep increasing stock exchanges. Only within one period the 1-month drift was the most successful. At the moderate course of the stock exchanges in this forecasting term, the forecasting accuracy was affected

DOI 10.12776/ams.v20i1.183 p-ISSN 1335-1532
e-ISSN 1338-1156

by the stock exchange with the initial condition drift. The advantage of 1-month drift consists in monthly shifting of the initial condition value to appropriate stock exchange for the next price evolution.

4 Discussion

From among 36 observed forecasting terms, the initial condition drift was noticed in 22 terms. The original forecasting fails within the periods with considerable changes of the price course. Within the observed period from 2003 to 2006, it was namely the period of the steep price decline in 2005 followed by a rapid price increase. Within the original forecasting there were 8 forecasting terms with the mean absolute percentage error $\geq 10\ \%$ (**Fig. 4**, **Fig. 5)**. Observing these terms after initial condition drift, only in two terms and just using 1-month drift, the mean absolute percentage errors of prognoses were $\geq 10\ \%$. However, if we consider the most successful type of drift in each term, all forecasting terms had the mean absolute percentage error $< 10\ \%$. When there were critical values in the forecasting term, their number was always reduced.

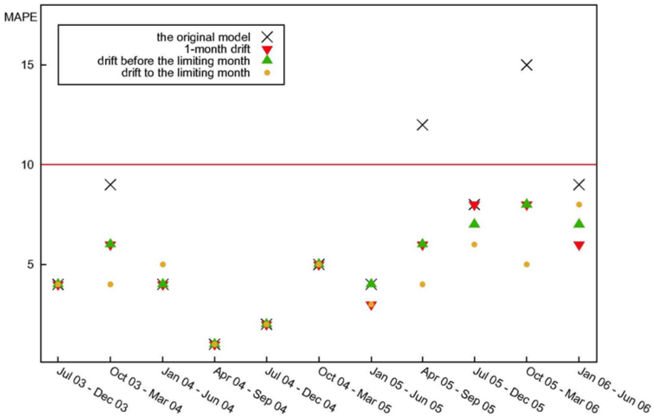

Fig. 4 The mean absolute percentage errors of the chosen types of the initial condition drift within the forecasting terms – variant B

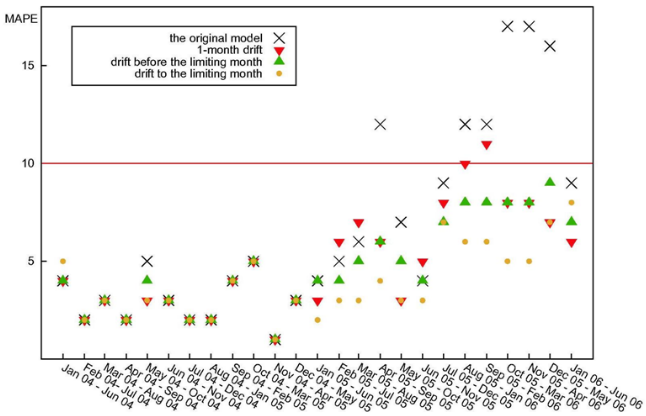

Fig. 5 The mean absolute percentage errors of the chosen types of the initial condition drift within the forecasting terms – variant E

DOI 10.12776/ams.v20i1.183 p-ISSN 1335-1532
e-ISSN 1338-1156

The most accurate prognoses were the most often obtained by the longest initial condition drift (**Table 1**, **Fig. 4** and **Fig. 5)**.

5 Conclusions

The original model forecasts the aluminium price reliably within the stable price course, but within the rapid increase or decrease of stock exchanges or in the case of changes in the price course the forecasting fails. With regard to the chosen results, the strategy of the initial condition drift significantly contributes to prognoses accuracy, and it is suitable way of the original forecasting improvement. The most accurate prognoses were the most often obtained by the longest initial condition drift. By using it, calculated prognoses were moved closed to the real stock exchanges. The initial condition drift to the limiting month allowed calculating the most accurate prognoses, especially in the stable increase up to the rapid increase, or during the longer-range decline in price. When the price fluctuations appeared in the observed period, the longest drift was not always the most accurate. The most accurate was the drift replacing the initial condition value by stock exchange that was the nearest to the next price course.

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DOI 10.12776/ams.v20i1.183 p-ISSN 1335-1532
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Acknowledgement
The research for this article was supported by Slovak VEGA Grant 1/0130/12.