

MATHEMATICAL MODELING OF SURFACE INDUCTION HEATING

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Abstract

The paper presents possibilities of controlling temperature field distribution in induction heated charge. The change of the distribution was obtained with use of sequential two-frequency heating. The study was conducted as multi-variant computer simulations of strongly coupled to each other field: the electromagnetic and the temperature fields. For the analysis, the professional calculation package using the finite element method Flux 3D was used. The problem of obtaining an appropriate temperature distribution in a heated charge of, a complex shape is very important in many practical applications. A typical example is a quenching of gears. For such applications, it is required to obtain a surface and in desired depth, uniform temperature distribution on the tooth face and top land and on the bottom land of the gear. The obtained temperature should have proper distribution and value. Achieving such a defined distribution is very difficult. During the study over a dozen different calculation variants were examined.

Keywords: quenching, induction heating, gears, coupled electromagnetic-thermal calculation

1 Introduction

Induction hardening is a metallurgical process whose purpose is to bring about local changes in the crystalline structure of surface layers of steel bodies resulting in their higher hardness [1–5]. The required parts of the body are first heated somewhat above temperature A_{c3} securing their uniform austenite internal structure. Then, after eventual equalization of temperatures, the body is intensively cooled by a suitable quenchant. The result is harder, but brittle martensite structure of the hardened parts. The structure of the rest of the body (its core) remains unchanged [6-9]. The situation is indicated in **Fig. 1**.

The figure shows several curves of cooling of typical carbon steel (41Cr4) from a starting temperature exceeding A_{c3} . Hardness (here in Vickers) HV is a function of the time t of cooling. The higher the velocity of cooling, the harder structure we obtain (see the curves on the left). The other curves (on the right) pass, however, through the area of bainite, pearlite or ferrite [10-12].

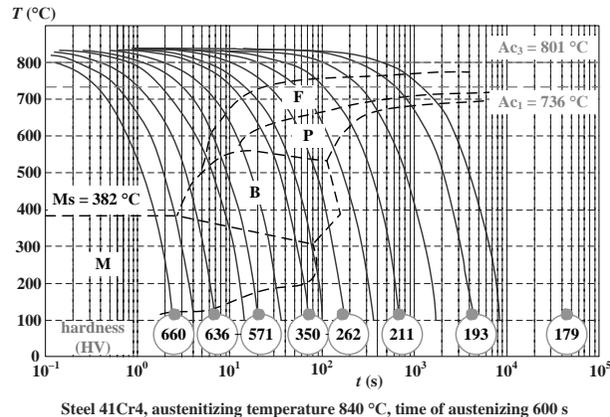


Fig. 1 CCT diagram of typical carbon steel 41Cr₄ (AISI 5140); M – martensite, B – bainite, P – pearlite, F – ferrite, Ac₃ – at this temperature steel exhibits uniform austenite structure Ms – temperature of the transition to martensite structure

The paper deals with the computer modelling of induction hardening of a teeth wheel. Its purpose is to harden their surfaces in order to increase their wear resistance [13]. The scheme of the arrangement is in **Fig. 2**. Real view of gear is in **Fig. 3**.

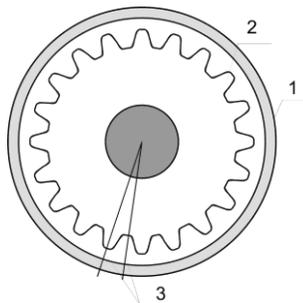


Fig. 2 Induction hardening of a teeth wheel, 1 – inductor, 2 – gear wheel, 3 - axes of symmetry



Fig. 3 Real view of a teeth wheel

2 Mathematical Model

The mathematical model of the problem is given by two partial differential equations describing distributions of harmonic electromagnetic field and nonstationary temperature field [14-20].

$$\operatorname{rot} \frac{1}{\mu} \operatorname{rot} A + \gamma (\mathbf{v} \times \operatorname{rot} A) = J_z \quad (1)$$

where: A - denotes vector potential V·s/m,
 γ - the electrical conductivity S/m,
 μ - the magnetic permeability H/m,
 J_z - uniform density of the external currents A/m²,
 \mathbf{v} - the relative speed of the inductor with respect to the charge m/s.

This equation must be supplemented with a correct boundary condition – in this case of the Dirichlet or Neumann type.

Distribution of nonstationary temperature field in the wheel is given by the heat transfer equation [4]

$$\operatorname{div}(\lambda \operatorname{grad} T) - \rho c(v \operatorname{grad} T) - \rho c \frac{\partial T}{\partial t} = -p_v \quad (2)$$

where: λ - denotes the thermal conductivity of the material W/(m · K),

ρ - its specific mass kg/m³,

c - its specific heat at a constant pressure (all previous coefficients being functions of temperature) J/kg K,

p_v - the specific average Joule in the material losses given as W/m³.

$$p_v = \frac{\mathbf{J} \cdot \mathbf{J}}{\gamma} \quad (3)$$

where: \mathbf{J} denotes the density of eddy currents induced in the heated wheel A/m².

$$\mathbf{J} = -\gamma \frac{\partial \mathbf{A}}{\partial t} \quad (4)$$

This is given by the formula.

The boundary conditions have to include convection and radiation.

The 3D computation of the problem was realized by the finite element method, using the code FLUX3D supplemented with a number of own procedures and scripts. A considerable attention was paid to selected numerical aspects of the solution, mainly to the convergence of results in the dependence on the density of discretization meshes for both magnetic and temperature field and also on the position of the artificial boundary in the case of magnetic field [21-24].

3 Results

The hardened wheel for automotive industry with 20 teeth is made of steel 41Cr4 (see **Fig. 1**). Its principal dimensions are given in **Table 1** and its thickness is 6 mm. The Curie temperature of steel 41Cr4 $T_C = 760$ °C, the austenitizing temperature $A_{C3} = 801$ °C [10].

Table 1 Principal dimension of gear

Symbol	Dimension [mm]	Designation
ϕ_1	30	internal diameter the gear
ϕ_2	94	external diameter the gear
ϕ_3	104	diameter between the teeth
ϕ_4	114	internal diameter of inductor

The physical parameters (as functions of the temperature) follow:

$$\frac{1}{\gamma(T)} = \frac{1}{\gamma_0} (1 + \alpha T) \quad (5)$$

where: $\gamma_0 = 3,3 \cdot 10^6 \text{ S/m}$
 $\alpha = 0,358 \cdot 10^{-6} \text{ K}^{-1}$

$$\lambda(T) = \lambda_0(1 + \beta T) \quad (6)$$

where: $\lambda_0 = 33 \text{ Wm}^{-1}\text{K}^{-1}$
 $\beta = 0$

$$\rho c(T) = \frac{E}{\sqrt{2\pi\sigma}} e^{-b} + (V_0 - V_i) e^{-\frac{T}{\tau}} + V_i \quad (7)$$

where: $E = 6 \cdot 10^8 \text{ J/m}^3$,
 $b = \frac{1}{2} \left(\frac{T - T_c}{\sigma} \right)^2$,
 $\sigma = 20 \text{ }^\circ\text{C}$,
 $V_0 = 0,38 \cdot 10^7 \text{ J/m}^3/^\circ\text{C}$,
 $V_i = 0,57 \cdot 10^7 \text{ J/m}^3/^\circ\text{C}$,
 $\tau = 550 \text{ }^\circ\text{C}$,

$$B(H, T) = \mu_0 H + \frac{2J_{s0}}{\pi} \arctg \left(\frac{\pi(\mu_{r0} - 1)\mu_0 H}{2J_{s0}} \right) \cdot f(T) \quad (8)$$

where: $J_{s0} = 2 \text{ T}$,
 $\mu_{r0} = 600 \text{ H/m}$,

$$f(T) = 1 - e^{-\frac{T - T_c}{C}} \quad \text{for } T \neq T_c \quad (9)$$

$$f(T) = 10e^{-\frac{T_2 - T}{C}} \quad \text{for } T \cong T_c$$

where: $C = 0,4$,
 $\alpha_c = 15 \text{ Wm}^{-2}\text{K}^{-1}$,
 $\varepsilon = 0,7$ (emissivity).

First, the field current in the inductor is set to 4500 A at the frequency $f = 10 \text{ kHz}$. At the moment when the average temperature of the tooth root reaches $1000 \text{ }^\circ\text{C}$, the current source is switched and the final part of heating is realized by the field current 1300 A at the frequency $f = 100 \text{ kHz}$.

The total time of heating is approximately 10.8 s, the time of cooling is 10 s.

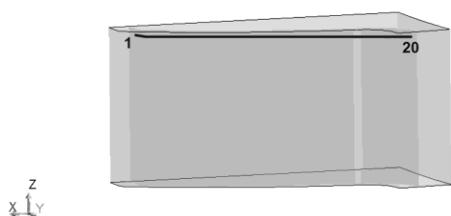


Fig. 4 The solved part of the arrangement

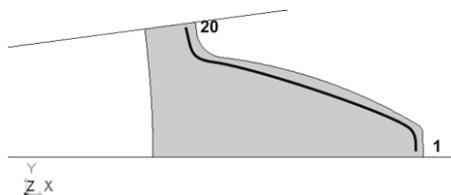


Fig. 5 Selected points of the tooth

Due to the symmetry, it is sufficient to analyze the part of the wheel according to **Fig. 4** (containing one half of the tooth). The thickness of this part is 3 mm (one half of the thickness of the wheel).

The number of DOFs for electromagnetic computations was about 26000, for thermal computations about 5000. The computation of one example took approximately 8 hours.

Fig. 5 shows a detail of the tooth with 20 points A1–A20, at the medium plane of the tooth where we checked the time evolution of the temperature.

Fig. 6 shows the profiles of the temperature at points A1–A11 at selected time steps. It is obvious that its distribution along the particular time levels is fairly homogeneous. The pictures (**Fig. 7–9**) show final magnetic induction, Joule (Power) Losses and temperature distribution on the tooth surface.

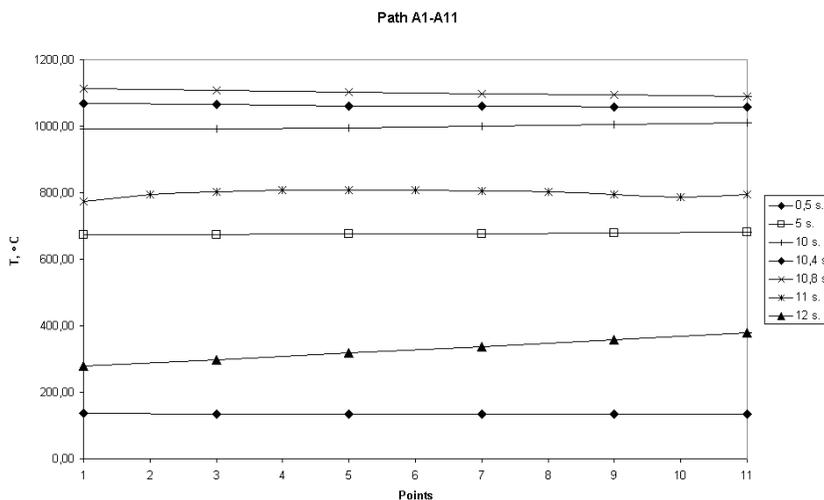


Fig. 6 Time evolution of temperature at points (A1-A11)

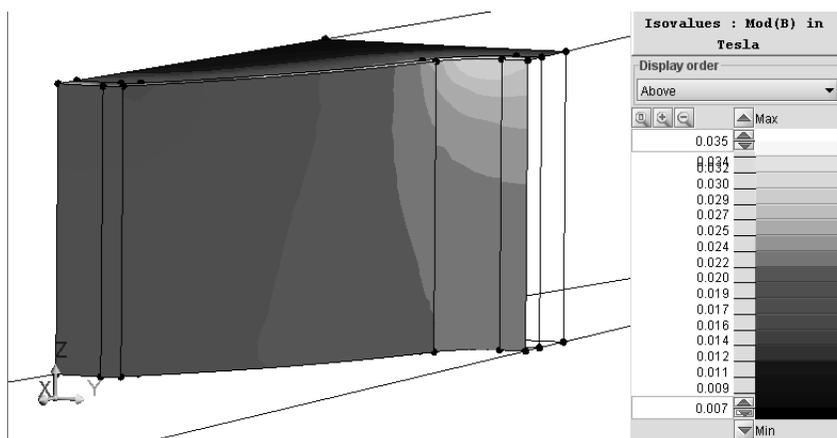


Fig. 7 Magnetic induction distribution on the tooth surface

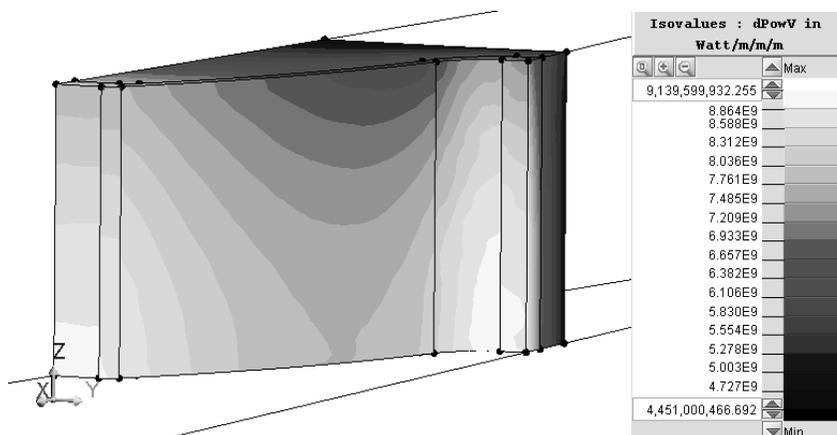


Fig. 8 Joule (Power) losses distribution on the tooth surface

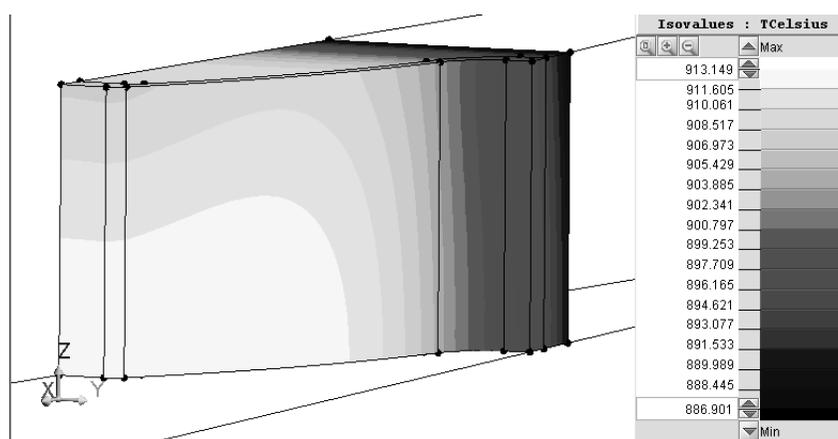


Fig. 9 Temperature distribution on the tooth surface after 10,8 s.

4 Discussion

This work presents numerical simulations of the induction heating. Such simulations contain the analysis of coupled electromagnetic and thermal fields. Analysis of the induction heating process of gear wheels seems to be a very complex process. This is mainly caused by the large number of parameters that have a decisive influence on the final result. In addition, the necessity of multiple calculations and changing the selected parameter to achieve the assumed temperature causes the large number of calculations.

5 Conclusions

The main conclusions are the following:

- (1) during the test quenching of the gear a problem of non uniform temperature distribution appears,
- (2) the attempts of experimental selection of heating parameters failed, and the problem was solved by multi-variant calculations,

- (3) the results obtained from calculations were successfully applied in the quenching process,
- (4) as the time of cooling below the martensite temperature is about 10 s, the resultant hardness of the surface layers of the tooth is about 641 HV.

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